

When Better Information Makes Protests Worse: Strategic Pricing and the Collapse of Information Aggregation*

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Abstract

How does market power distort information aggregation through collective action? A monopolist's price screens which consumers remain in the market, shaping what the firm learns from protest turnout. We identify two competing forces: a higher price raises each participant's pivotal impact but homogenizes participation incentives. When individual signals are precise, the latter dominates, and the profit-maximizing price destroys protest informativeness. This yields a *Blackwell paradox under endogenous pricing*: more informative individual signals lead to less informative collective outcomes. Our findings provide a new rationale for price regulation: preserving the informational role of consumer activism.

Keywords: Protest, information aggregation, monopoly pricing, Blackwell paradox

JEL Classification: D42, D72, D81, D82

1. Introduction

A central question in information economics is whether decentralized agents can aggregate dispersed information through collective action. A large literature, originating with Lohmann (1993, 1994), has shown that protests, demonstrations, and other forms of collective expression can serve as effective information aggregation devices when directed at governments or other non-market decision-makers. In these settings, a key comparative static is straightforward: more precise individual signals improve the informativeness of collective action. This paper shows that this comparative static breaks down when collective action targets a price-setting firm, identifying a new channel through which market power distorts information aggregation.

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The mechanism is as follows. A monopolist's price determines which consumers participate in the market, and this selection in turn determines what the firm can learn from consumer protests. We decompose the effect of prices on protest informativeness into two forces. A higher price can raise each remaining consumer's pivotal impact on the firm's continuation decision (a *pivotality effect*), but it also compresses the surplus that buyers expect from the product's continued availability. Because this surplus is the opportunity cost of protesting for removal, its compression narrows the gap between buyers' and non-buyers' protest incentives, making the protest less informative (an *information-rent effect*). When individual signals are precise, the information-rent effect dominates: profit-maximizing firms set prices that endogenously destroy protest informativeness.

This yields what we term a *Blackwell paradox under endogenous pricing*: a Blackwell-more-informative individual signal structure can produce a Blackwell-less-informative collective outcome once the firm optimally chooses its price. In models of protests against governments (Ekmekci and Lauermann, 2022), individual signal precision unambiguously improves information aggregation. In our setting, the opposite occurs. The difference stems entirely from the firm's ability to screen its customer base through pricing.

To fix ideas, consider the following examples. Consumers have organized campaigns against Nike over labor violations;¹ Adidas withdrew a product line following social media backlash;² and publishers have faced pressure to cancel controversial books.³ Yet the success of these campaigns in changing firm behavior varies widely. Our model provides one explanation: the firm's pricing strategy can endogenously neutralize the information that protests are designed to convey. Suggestive evidence comes from the 2020 Goya Foods boycott, where the firm maintained or raised prices amid widespread consumer backlash, profiting from a smaller pool of committed buyers and limiting the campaign's informational impact (Liaukonytė et al., 2023).

We formalize this in a model where a monopolist sells a product whose value to consumers is uncertain. This value may reflect psychological factors such as guilt from purchasing a controversial product, reputational concerns, or evolving social norms about the product's acceptability. Each consumer receives a private signal about this value and may incur a cost to join a protest campaign demanding the product's removal. After observing a noisy public signal of the protest's scale, the firm decides whether to continue selling. The firm also bears a cost of continuation, capturing distribution expenses and reputational risk, so it benefits from learning whether its product is truly valued by consumers.

¹ E.g., see "Protesters call on NYU Bookstore to cut ties with Nike," *Washington Square News*, October 13, 2023.

² E.g., see "Adidas cancels 'shackle' shoes after outcry," *CNN*, June 20, 2012.

³ E.g., see "Japan firm nixes translation of U.S. book questioning trans surgery," *Japan Times*, December 6, 2023.

We first establish the price–informativeness trade-off for a general continuous-signal environment (Theorem 1). In low-noise environments, the information-rent effect dominates, yielding a *price–informativeness trade-off*: higher prices reduce protest informativeness. Importantly, this trade-off is not universal. We show that under some conditions, the comparative statics reverse in high-noise environments: higher prices increase informativeness, because the pivotality effect prevails. The fact that the direction depends on the informational environment confirms that the result is driven by the interaction of the two forces, not by a mechanical consequence of screening alone.

We then specialize to binary signals to characterize the full equilibrium with endogenous pricing. Informative protest equilibria exist only at intermediate prices (Proposition 2). When signal precision is high, the firm finds it profitable to set a price that targets only consumers with favorable signals, rendering the protest uninformative. When signal precision is low, the firm sets a lower price to sell to a broader base, and the resulting heterogeneity in purchase incentives sustains informative protests (Theorem 2). More precise consumer information can therefore reduce social welfare by inducing the firm to choose prices that eliminate the protest’s informational content (Corollary 1). We also show that the firm’s expected profit is increasing in protest informativeness (Proposition 7), yet the gain from surplus extraction outweighs the informational loss: the firm optimally destroys the very informativeness from which it benefits.

We further analyze an uncommitted pricing regime where the firm sets the price after observing the protest signal. Here, an additional force emerges: consumers may strategically participate in protests to manipulate the firm’s subsequent pricing, since higher turnout signals lower product value and induces price reductions. This price-manipulation motive⁴ is stronger for high-signal consumers, who benefit more from price reductions because they are more likely to purchase. When signal precision is high, this motive reverses the natural participation pattern, rendering the protest uninformative. The central conclusion thus extends to the uncommitted case: the Blackwell paradox under endogenous pricing persists regardless of the pricing regime.

Our results yield concrete policy implications. Because informative protest equilibria exist only within an intermediate price window, modest price-cap policies that keep prices inside this window can improve welfare by preserving the disciplinary and informational role of consumer activism. Under uncommitted pricing, improving the accuracy and transparency of protest reporting serves as a policy substitute for direct price regulation, because precise public signals of turnout restore the pivotal-information effect and suppress the price-manipulation motive.

⁴ This motivation is analogous to strategic waiting in dynamic pricing (Su, 2007) and demand reduction in multi-unit auctions (Ausubel et al., 2014).

While our formal analysis focuses on consumer protests, the core mechanism—that pricing screens the feedback pool and compresses heterogeneity in feedback incentives—can extend to online reviews (Li and Hitt, 2008; Acemoglu et al., 2022) and shareholder activism (Levit et al., 2024). We discuss these applications in Section 6.

The rest of the paper is organized as follows. The remainder of this section discusses related literature. Section 2 presents the model. Section 3 analyzes the equilibrium with continuous signals. Section 4 focuses on binary signals and endogenizes price-setting behavior. Section 5 considers uncommitted pricing. Section 6 discusses modeling assumptions and concludes.

1.1. Related Literature

Our paper contributes to three literatures and connects them in a new way.

Protests as information aggregation: A central reference is the literature on protests as information aggregation devices (Lohmann, 1993, 1994; Battaglini, 2017; Ekmekci and Lauermann, 2022). In these models, citizens campaign against a government whose policy is the sole decision variable, and individual signal precision unambiguously improves protest informativeness. Our setting introduces a price-setting firm, which fundamentally changes the comparative statics. The key difference is that the firm’s price endogenously determines the composition of the protesting population, a channel that is absent when the decision-maker does not set prices. Our Blackwell paradox, that more precise individual signals can reduce collective informativeness, is therefore distinctive to markets with endogenous pricing.

Consumer boycotts and corporate activism: Existing models of boycotts study them as public goods games (Diermeier and Van Mieghem, 2008), wars of attrition (Egorov and Harstad, 2017; Correa, 2025), or signaling devices (Feddersen and Gilligan, 2001; Innes, 2006; Miyagiwa, 2009; Glazer et al., 2010; Heijnen and Made, 2012; Peck, 2017). These studies treat the firm’s decision environment as largely separate from consumers’ protest incentives. By contrast, we emphasize the feedback loop between pricing and protest participation: the firm’s price determines who is in the market, which in turn determines what the firm can learn from collective action.

Broccardo et al. (2022) compare exit (divestment and boycott) with voice (shareholder engagement), showing that voice is more effective when socially responsible agents are in the majority. Our analysis identifies a channel through which voice can fail that is absent from the exit-versus-voice framework: the firm’s pricing strategy endogenously undermines the informativeness of voice. Specifically, in their model the information environment is exogenous to the firm’s actions, whereas in ours the firm’s price endogenously determines the information content of consumer activism. Kaufmann et al. (2024) show that equilibrium feedback in

competitive markets dampens individual mitigation efforts. Our model reveals a distinct channel: the firm's pricing strategy, rather than competitive feedback, undermines the effectiveness of collective consumer action.

Market power and information distortion: In limit-pricing models, incumbents set prices to influence potential entrants' inferences (Milgrom and Roberts, 1982; Bagwell and Ramey, 1988), and Goldstein and Guembel (2008) show how manipulation can reduce the informational efficiency of financial prices. Our mechanism is distinct: the firm's pricing manipulates the informativeness of a separate social signal (protest turnout) by screening participants and compressing information rents.

Our paper also relates to the broader literature on information design (Kamenica and Gentzkow, 2011; Bergemann and Morris, 2019), but with a fundamental distinction. In Bayesian persuasion, a sender directly chooses the information structure. Unlike Bayesian persuasion, the distortion in our model is a byproduct of surplus extraction rather than a deliberate design choice: no agent designs the information environment, but the firm's price *inadvertently* shapes the informativeness of a social signal.

2. Model

A firm sells a product to a continuum of consumers with unit demand. The population is normalized to 1. The utility of purchasing the product is $w \cdot v$, where $v \in \{0, 1\}$. We refer to v as *product value* and w as the scale parameter, whose role is discussed later. The value v can be interpreted as incorporating a psychological factor, such as feeling guilty about purchasing a product that causes a problem. We assume that v is unknown to all players. For instance, this uncertainty about v captures situations where a problem with the product has just been revealed, and people are uncertain about how to respond collectively. The prior probability of $v = 1$ is assumed to be $1/2$ for notational simplicity.

Consumers can engage in a protest campaign to stop the sale of the product. Participating in the campaign costs each consumer i an amount c_i . The c_i are i.i.d. with distribution function F on $\mathbb{R}$ among consumers (we allow negative costs). We assume that f is the density of F , and $f(0) > 0$. In summary, when the product is sold at price $w \cdot p$, the consumer's utility is as follows.⁵

$$u_i = [w \cdot (v - p) \cdot \chi(\text{purchase})] \cdot \chi(\text{on sale}) - c_i \cdot \chi(\text{participate in protest}).$$

Before participating in the protest campaign, consumers receive a private signal about the product value v . Consumers receive a private signal represented by the posterior belief $\theta_i \in$

⁵For each event E , $\chi(E)$ takes the value 1 if E occurs, and 0 otherwise.

$(0, 1)$, which denotes the conditional probability that $v = 1$. Formally, we define $\theta = \frac{g_1(\theta)}{g_1(\theta) + g_0(\theta)}$, where g_v and G_v denote, respectively, the density and cumulative distribution of the posterior under state v . By this specification, G_v has the monotone likelihood ratio property (MLRP): $g_1(\theta)/g_0(\theta)$ is non-decreasing in θ .

Given consumers' private signals, they decide whether to participate in the protest campaign. We assume that consumers behave according to a **rule-utilitarian** (or **group-utilitarian**) principle, choosing to protest if their participation increases the aggregate welfare of consumers who received the same signal. This framework, which follows Feddersen and Sandroni (2006) and Ekmekci and Lauermaun (2022), allows us to directly model the pivotal incentives central to collective action. The precise objective function is given in Section 3.1. We discuss the rule-utilitarian assumption further in Section 6.

One justification for this assumption is that a consumer's participation decision is observed by their social network, leading them to prefer being perceived as ethical. This, in turn, motivates them to act in a way that maximizes consumers' welfare.⁶

However, the adequacy of this justification depends on what participation in the protest entails. On the one hand, if participation involves attending an event organized by activists, the cost may be too high for this motivation to hold. On the other hand, if participation means sharing or liking a post on social media that expresses regret or protest against a firm's product, this motivation seems more plausible.⁷

After the protest, the firm and consumers observe a public signal indicating its scale. Let the actual participation ratio be τ . The firm and consumers observe $t = \tau + \varepsilon$, where ε is normally distributed noise with mean 0 and variance σ^2 . Let Φ be the standard normal distribution function and φ be its density. ε can be interpreted as noise from media coverage or ambiguity in measuring the protest's true size. For instance, it could reflect noise intentionally created by activists.⁸

Observing the public signal t , the firm decides whether to discontinue the product. We assume that the firm incurs a cost of $wK > 0$ when continuing product sales. This includes distribution and reputation costs associated with selling a product that causes a problem. We also assume that the marginal cost of production is $w\kappa \geq 0$. Therefore, when the product is on sale, the firm's profit is

$$w \cdot [(\text{demand at } p) \cdot (p - \kappa) - K].$$

⁶ Ali and Lin (2013) explores this rationale in the context of voting decisions.

⁷ Sharing and liking posts on social media can be enough to compel a firm to discontinue a product. For instance, Adidas withdrew a line of shoes resembling shackles in response to criticism on Facebook ("Adidas cancels 'shackle' shoes after outcry," *CNN*, June 20, 2012).

⁸ This assumption is the same as Ekmekci and Lauermaun (2022).

We normalize the firm's payoffs and write profits without the scale parameter w , as it does not affect the firm's decision. Hence, the firm's profit when the product is on sale is $(\text{demand at } p) \cdot (p - \kappa) - K$.

We assume that $1 > \kappa + K$ to exclude a trivial case. Otherwise, the firm cannot yield positive profit, so not selling is always optimal.

Finally, the decision flow is as follows. The solution concept is the *perfect Bayesian Nash equilibrium*.

1. The firm sets a price.
2. Each consumer observes a private signal θ_i and decides whether to participate in the protest.
3. Turnout τ is realized, and the firm and consumers observe public signal $t = \tau + \varepsilon$.
4. The firm decides whether to continue selling the product.
5. If the product is on sale, each consumer decides whether to buy it.

3. Equilibrium with a given price

3.1. Optimal behavior

This section characterizes the optimal behavior of consumers and the firm after the price is set. We first consider the consumer's purchase choice, then the firm's discontinuation decision, and finally the consumer's decision to participate in the protest.

Consumer's purchase decision: We first consider consumers' purchase decisions after public signal observation. Consider the consumer receiving signal θ . The consumer purchases if the expected product value exceeds the price, that is, $\Pr(v = 1 \mid \theta, t) \geq p$.⁹

For notational simplicity, we write $P_\theta(t) = \Pr(v = 1 \mid \theta, t)$. Let λ_v be the true turnout when the realized value is $v \in \{0, 1\}$ and $\lambda = \lambda_0 - \lambda_1$. Then, as $t = \lambda_v + \varepsilon$ at state v , $P_\theta(t)$ is written as follows (recall that φ is the pdf of the standard normal distribution).

$$P_\theta(t) = \frac{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta}{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta + \varphi\left(\frac{t-\lambda_0}{\sigma}\right)(1-\theta)}.$$

Using this formula, when we assume $\lambda_0 > \lambda_1$, inequality $P_\theta(t) > p$ is rewritten as

$$t < \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2\lambda} \left[\ln\left(\frac{\theta}{1-\theta}\right) - \ln\left(\frac{p}{1-p}\right) \right] =: T_\theta(p).$$

⁹Note that as this decision is in the last stage of the game, the purchase decision does not affect others' utilities. Thus, each consumer's purchase decision is based only on their utility, although the consumers have welfare concerns.

In other words, $T_\theta(p)$ is the supremum value of public signal t at which the type- θ consumers purchase the product. The following lemma shows that high- θ consumers are more likely to purchase the product than low- θ consumers.

Lemma 1. *If $\lambda_0 > \lambda_1$, $T_\theta(p)$ is increasing in θ for any $p \in (0, 1)$.*

By this lemma, $T_\theta(p)$ determines another cutoff: Let $\Theta_t(p)$ be the minimum θ such that $T_\theta(p) > t$. Then,

$$\Theta_t(p) = \psi \left(\frac{2\lambda}{\sigma^2} \left[t - \frac{\lambda_0 + \lambda_1}{2} \right] + \ln \left(\frac{p}{1-p} \right) \right),$$

where $\psi(x) = \exp(x)/(1 + \exp(x))$. Therefore, when the product value is v , the demand for the product is $1 - G_v(\Theta_t(p))$.

The firm's discontinuation decision: Given the consumer's choice after observing public signal t , this section considers the condition under which the firm discontinues the product sale. We focus on a cutoff equilibrium; there is a cutoff $T_f(p)$ such that the firm discontinues the product sale if and only if $t > T_f(p)$. To determine $T_f(p)$, let $\Pi(p, t)$ be the expected net-profit of the firm after observing public signal t :

$$\Pi(p, t) = \sum_{v \in \{0, 1\}} \Pr(v | t) [1 - G_v(\Theta_t(p))] (p - \kappa) - K.$$

Next, we show that the profit can be decomposed into (λ, t, σ) -dependent and (λ, t, σ) -independent parts. Let $a = \Pr(v = 0 | t) / \Pr(v = 1 | t) = \frac{\varphi((t - \lambda_0)/\sigma)}{\varphi((t - \lambda_1)/\sigma)} = \exp(\lambda \sigma^{-2} [t - (\lambda_0 + \lambda_1)/2])$ be the likelihood ratio of the product value conditioned on public signal t . Then, the profit is rewritten as

$$\Pi(p, t) = \Pi^*(p, a) := \frac{p - \kappa}{1 + a} \left[(1 - \tilde{G}_1(\omega(p, a))) + a(1 - \tilde{G}_0(\omega(p, a))) \right] - K,$$

where $\omega(p, a) = 2 \ln(a) + \ln(p/(1 - p))$ and $\tilde{G}_v = G_v \circ \psi$. In other words, a is a sufficient statistic for determining the profit. Let $a^*(p)$ be the solution of $\Pi^*(p, a) = 0$ with respect to a , which is independent of λ , t , and σ . Then, the firm's cutoff is written as follows: $T_f(p) = \sigma^2 \lambda^{-1} \ln(a^*(p)) + (\lambda_0 + \lambda_1)/2$.

Consumer's participation decision: Before observing the public signal, the consumer chooses whether to participate in the protest. This section examines the participation condition and protest informativeness.

We focus on the cutoff strategy: each type- θ consumer participates in the protest if and only if the marginal benefit of participation exceeds its cost. As mentioned in the model section, we consider consumers who follow a rule-utilitarian principle: they maximize the aggregate utility

of consumers of the same type, as specified by the following expression.^{10, 11}

$$\int_{T_f(p)-d\tau}^{T_f(p)} w \cdot [P_\theta(t) - p]_+ \cdot \xi_\theta(t) dt - c_i d\tau - \sum_{j \text{ is a participant}} c_j,$$

where we define $[\cdot]_+ = \max\{\cdot, 0\}$ and $\xi_\theta(t) = \sigma^{-1} \cdot [\theta \cdot \varphi((t - \lambda_1)/\sigma) + (1 - \theta) \cdot \varphi((t - \lambda_0)/\sigma)]$. Here, $\xi_\theta(t)$ is the probability of receiving public signal t conditioned on signal θ . The term $d\tau$ represents a marginal increase in the participation rate. In this formula, the firm applies a cutoff criterion: discontinuing the product if and only if $t > T_f(p)$. A consumer's participation increases turnout by $d\tau$, which also increases their costs by $c_i d\tau$. Then, consumer i participates in the protest if and only if

$$\underbrace{-w \cdot [P_\theta(T_f(p)) - p]_+ \cdot \xi_\theta(T_f(p))}_{\text{marginal benefit of participation}} \geq c_i.$$

Now let $c_\theta(p) = -w[P_\theta(T_f(p)) - p]_+ \cdot \xi_\theta(T_f(p))$ be the cutoff for a type- θ consumer. The next subsection characterizes the equilibrium and performs comparative statics.

3.2. Equilibrium and protest informativeness

Given the optimal behavior characterized above, this subsection studies the continuation-game equilibrium at a given price p . Then, we define and analyze the informativeness of the protest.

In the previous subsection, to determine the optimal behavior, the value $\lambda = \lambda_0 - \lambda_1$ was exogenously given. We endogenize this value. As the cutoff for the participation decision is $c_\theta(p)$, λ_v , the participation rate when the product value is v is calculated as

$$\lambda_v = \int F(c_\theta(p)) g_v(\theta) d\theta.$$

As $c_\theta(p)$ depends on λ , define $\Lambda(\lambda, p)$ as follows:

$$\Lambda(\lambda, p) = \lambda_0 - \lambda_1 = \int F(c_\theta(p)) (g_0(\theta) - g_1(\theta)) d\theta.$$

The equilibrium under price p is characterized as a fixed point of $\Lambda(\cdot, p)$, that is, $\lambda^*(p) = \Lambda(\lambda^*(p), p)$.

We refer to λ^* as the **informativeness** of the protest.¹² Indeed, we have¹³

Fact 1. *The distribution of ex post belief $\{P_\theta(t)\}_{t \in \mathbb{R}}$ with higher λ is a mean-preserving spread of that with lower λ .*

We say that the protest equilibrium is *uninformative* if $\lambda^*(p) = 0$ and *informative* if $\lambda^*(p) > 0$.

¹⁰ Note that assuming this utility does not affect the purchasing decision, as it does not affect the other consumers' utility and the firm's decision.

¹¹ Our results continue to hold even when each consumer maximizes overall consumer welfare. See Section 6.

¹² This definition is the same as Ekmekci and Lauermaun (2022).

¹³ Although this fact is standard, we provide a proof in Appendix B for completeness.

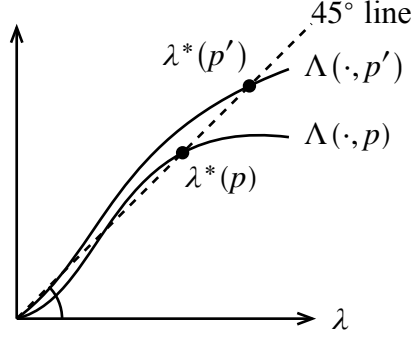


Figure 1: $\lambda^*(p)$ indicates the largest fixed point of $\Lambda(\cdot, p)$

An uninformative equilibrium exists for all parameter values. If the firm sets $T = \infty$ or $T = -\infty$, the protest does not affect whether the product sale continues. This leads to both types of consumers having indifferent incentives. Conversely, if the protest is uninformative, the firm's decision remains unchanged regardless of the turnout. This implies that $T = \infty$ or $T = -\infty$.

Therefore, we focus on the most informative equilibria to obtain clear comparative statics. In particular, to focus on the most informative equilibrium, we consider the largest fixed point of $\Lambda(\cdot, p)$.

As we focus on the largest fixed point, if $\Lambda(\lambda, p)$ is (weakly) increasing in p pointwise, then $\lambda^*(p)$ is non-decreasing in p ; likewise, if $\Lambda(\lambda, p)$ is (weakly) decreasing in p , then $\lambda^*(p)$ is non-increasing in p (see Figure 1 for an illustration).¹⁴

The price can reduce protest informativeness. The following theorem provides a sufficient condition under which higher prices reduce Λ .

Theorem 1 (Price–Informativeness Tradeoff). *Let $\eta_a(p) := \frac{\partial a^*(p)/\partial p}{a^*(p)}$. $\partial \Lambda(\lambda, p)/\partial p < 0$ for sufficiently small σ if*

$$(1 - a^*(p)) \left[1 + \left[\frac{p - \frac{1}{2}}{a^*(p) - 1} + \frac{p}{2} \right] \eta_a(p) \right] > 0.$$

This result captures the two competing forces described in the Introduction. To see the intuition, recall that the participation cutoff decomposes as:

$$c_\theta(p) = -w \underbrace{[P_\theta(T_f(p)) - p]_+}_{\text{Information rent}} \cdot \underbrace{\xi_\theta(T_f(p))}_{\text{Pivotal prob.}}.$$

The price affects participation in two ways: one is on the *information rent*, and the other is on the probability of being *pivotal*.

The pivotal effect is determined by how the price affects the firm's cutoff $T_f(p)$, and this effect can be either positive or negative. In contrast, the effect on information rent is $w[1 -$

¹⁴ This fact is standard. Furthermore, this fixed point is stable under adaptive dynamics.

$P_\theta(1 - P_\theta)\eta_a(p)]$, which decreases with $\eta_a(p)$, the elasticity of a^* . The sufficient condition given in Theorem 1 specifies when the information-rent effect dominates the pivotal effect. In this case, λ decreases because the difference in c_θ is compressed.

The price–informativeness trade-off is not universal. Under some conditions on the firm’s profit structure, the comparative statics reverse in high-noise environments: higher prices *increase* informativeness, because the pivotality effect prevails. The fact that the direction of the relationship depends on the informational environment confirms that the result is driven by the interaction of the two economic forces, not by a mechanical consequence of screening alone. Appendix C provides the formal statement and sufficient conditions for the high-noise case.

Finally, we consider the optimal price that maximizes the total profit. Fix λ and define

$$\Pi_\lambda(p) = \frac{1}{2\sigma} \int_{-\infty}^{T_f(p)} \Pi(p, t) \left[\varphi\left(\frac{t-\lambda_1}{\sigma}\right) + \varphi\left(\frac{t-\lambda_0}{\sigma}\right) \right] dt.$$

Then, $p_\sigma^* = \arg \max \Pi_\lambda(p)$ is the optimal price when the effect on λ is ignored.

Lemma 2. *Suppose that λ is given. Let $p_a = \arg \max_p \Pi^*(p, a)$. Then, when $\sigma \rightarrow 0$ or $\sigma \rightarrow \infty$, p_σ^* converges to $p_{a_\sigma^\dagger}$ where $a_\sigma^\dagger = \exp(\lambda/\sigma)$.*

To shorten notation, $p_\sigma^\dagger := p_{a_\sigma^\dagger} = \arg \max_p \Pi^*(p, a_\sigma^\dagger)$. By this lemma, to know $\partial \Lambda / \partial p$ in a high or low noise environment, we only need to consider $a^*(p_\sigma^\dagger)$ and apply Theorem 1.

In a low-noise environment (i.e., $\sigma \rightarrow 0$), we show that $p_{a_\sigma^\dagger} \rightarrow 1$. Intuitively, when noise is small, the protest almost perfectly reveals the realized value v . If $v = 1$, setting $p \approx 1$ is optimal. If $v = 0$, not selling the product is optimal. In either case, setting $p \approx 1$ is approximately dominant. Under some conditions, we show that price and protest informativeness conflict: the optimal price should be lower when the firm prefers more informative protests.

Proposition 1. *Suppose that $\tilde{g}_v(x) > 0$ and $\lim_{x \rightarrow \infty} (1 - \tilde{G}_v(x)) / \tilde{g}_v(x) = \infty$.¹⁵ Then, $\lim_{\sigma \rightarrow 0} \partial \Lambda(\lambda, p) / \partial p|_{p=p_\sigma^\dagger} < 0$.*

The vanishing hazard rate of $\tilde{G}_v$ means that the upper tail of the signal distribution is thick: as $p \rightarrow 1$, a nonnegligible mass of consumers has θ close to 1. For these high types, the information rent $P_\theta(T_f(p)) - p$ is small and varies little across types. This compression of information rents strengthens the information-rent effect relative to the pivotal effect, so that the sufficient condition in Theorem 1 is satisfied.

Notably, this conflict is not because the firm dislikes informative protests. Proposition 7 in Appendix D shows that, in low-noise environments, the firm’s expected profit is increasing in λ : more informative protests help the firm make better discontinuation decisions. The tension

¹⁵ The condition $\tilde{g}_v > 0$ ensures full support. E.g., $\tilde{G}(x) = 1 - 1/x$ for large x satisfies both conditions.

arises because the profit-maximizing price endogenously destroys the very informativeness from which the firm would benefit.

4. Full-equilibrium characterization with binary signals

From this section onward, we assume a binary signal to characterize the full equilibrium. That is, G_v has a Dirac mass on $\{\theta_H, \theta_L\}$. Specifically, we assume that $P(\theta_i = \theta_H | v = 1) = P(\theta_i = \theta_L | v = 0) = \mu \in (1/2, 1)$. We refer to μ as *signal precision*. Then, the posteriors are $\theta_H = \mu$ and $\theta_L = 1 - \mu$.

The binary-signal model connects to the continuous-signal analysis of Section 3 as follows. The price–informativeness trade-off (Theorem 1) applies to any signal distribution; the binary case allows us to characterize the equilibrium in closed form and derive sharp comparative statics in μ .

4.1. Equilibrium characterization

By the binary-signal simplification, we provide the explicit form of $T_f(p)$. Hereafter, to simplify the notation, $T_{\theta_H} = T_H$, $c_{\theta_H} = c_H$, and we refer to a type- θ_H consumer as a type- H consumer. The same simplification is applied to θ_L . Now the following lemma characterizes the firm's cutoff.

Lemma 3. *Suppose that $T_L(p) < T_H(p)$, and let*

$$\tilde{T} := \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)}{\lambda_0 - \lambda_1}.$$

Then, the cutoffs are classified as follows.

- (i) *If $p - \kappa \geq K/(1 - \mu)$, $T_f(p) = T_H(p)$.*
- (ii) *If $K/(1 - \mu) > p - \kappa > K/\mu$, $T_f(p) = \max\{\min\{T_H(p), \tilde{T}\}, T_L(p)\}$.*
- (iii) *If $K/\mu \geq p - \kappa \geq K$, $T_f(p) = T_L(p)$.*
- (iv) *If $K > p - \kappa$, $T_f(p) = -\infty$.*

Our discussion is guided by the following proposition, which characterizes the range of prices for which an informative equilibrium exists. This result is the key building block for the Blackwell paradox.

Proposition 2. *Let $\bar{p} := \mu(K + (1 - \mu)\kappa)/(\mu(1 - \mu) + (2\mu - 1)K)$. (a) If $p \geq \bar{p}$ or $p \leq K + \kappa$, no informative equilibrium exists. (b) Suppose that $K + \kappa < p < \bar{p}$, $f(c) > 0$ for any $c \in \mathbb{R}$, and $F(0) > 0$. Then, if w is sufficiently large, an informative equilibrium exists. Moreover, the equilibrium $\lambda^* \geq (2\mu - 1)F(0) - \delta$ for arbitrary $\delta > 0$ by taking large w .*

An important consequence of Proposition 2 is that it reveals a fundamental commitment problem faced by the firm: for the equilibrium to be informative, the price should be below $\bar{p}$. This price ceiling reveals a crucial tension. Even if a protest could perfectly reveal the state (e.g., $v = 1$), the firm cannot fully exploit this information by setting a price close to 1. The reason is that the protest's informativeness is endogenous: the very act of setting a high price would, by the logic of Proposition 2, render the protest uninformative in the first place. This dynamic has a perverse consequence for consumers. Note that the price ceiling for an informative protest, $\bar{p}$, increases with signal precision μ if, for example, $\mu > \sqrt{\kappa + K}$.¹⁶ In such a case, more precise private information does not necessarily benefit consumers; it may simply empower the firm to maintain a higher price.

4.2. Firm's optimal pricing

Assuming that for any given price, the most informative equilibrium prevails, this section endogenizes the firm's price choice to determine the overall equilibrium outcome.

First, as a benchmark, we consider the model without protest. In the benchmark case, the type- H 's expected product value is $E[v | \theta_H] = \mu$, and that of the type- L consumers is $E[v | \theta_L] = 1 - \mu$. If the firm continues selling the product, the optimal price is $p = \mu$ or $p = 1 - \mu$. The expected profit when $p = \mu$ is $(\mu - \kappa)/2 - K$, and that when $p = 1 - \mu$ is $1 - \mu - \kappa - K$.

Now, we consider the firm's optimal pricing with the protest. The following proposition shows that the optimal price is $p = \mu$ when μ is high enough, in which case the protest is uninformative.

Proposition 3. *Suppose that $\mu > \kappa + 4K$, $\kappa + K > 1/2$ and $M(\mu) > 0$, where*

$$M(\mu) := \mu - \kappa - K \frac{\mu + \kappa(2\mu - 1)}{\mu(1 - \mu) + (2\mu - 1)K} - K.$$

Then, $p = \mu$ is optimal and the resulting equilibrium is uninformative.

As illustrated in Figure 2, $M(\mu) > 0$ is possible for intermediate μ when K is small.

The intuition for this result hinges on the firm's trade-off between information acquisition and surplus extraction. As Proposition 2 shows, informative equilibrium requires the price to be below $\bar{p}$. The condition $M(\mu) > 0$ ensures that $\bar{p}$ is small enough, in which case setting $p = \mu$ —which yields an uninformative equilibrium—generates greater profit. The information is valuable for the firm when K is high. Therefore, the uninformative equilibrium with a high price is preferable when K is low.

¹⁶ Indeed, by a differentiation, $\bar{p}$ is increasing in μ if and only if $-2\kappa\mu(1 - \mu) - \mu^2 + \kappa + K < 0$.

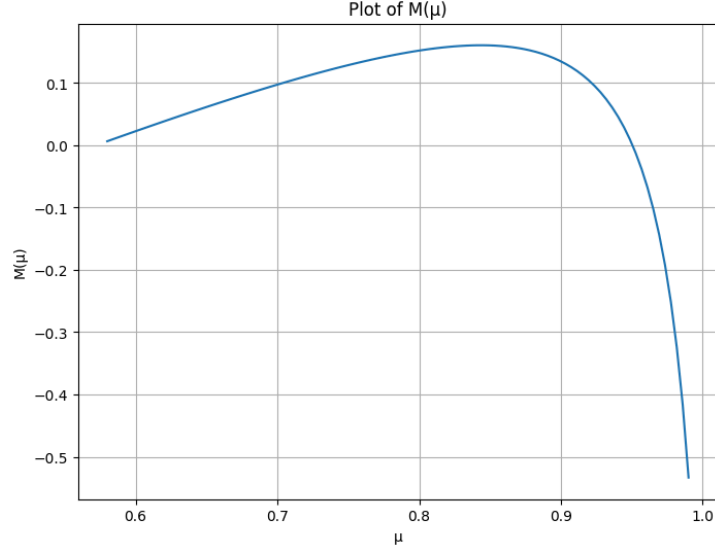


Figure 2: Graph of $M(\mu)$ when $\kappa = 1/2$, and $K = 0.02$

The above observation depends on whether an uninformative equilibrium with $p = \mu$ yields a sufficient profit. If not, an informative equilibrium leads to a higher profit, as the following proposition shows.

Proposition 4. *Suppose that $\mu \in (K + \kappa, 2K + \kappa)$. Then, for sufficiently small σ , there is $\bar{w}$ such that for each $w > \bar{w}$, the protest is informative at the firm's optimal price.*

Under the conditions of Proposition 4, $\bar{p} > \mu$, and the uninformative equilibrium yields a deficit. Therefore, setting the price to $p = \bar{p}$ induces an informative equilibrium that yields a positive profit.

By Propositions 3 and 4, we observe that the resulting equilibrium is uninformative for a higher μ , while it is informative for a lower μ . Theorem 2 summarizes the result.

Theorem 2. *Suppose that $\kappa + K > 1/2$. For $\mu \in (K + \kappa, 2K + \kappa)$, an informative equilibrium exists for sufficiently large w and small σ . For $\mu > 4K + \kappa$ with $M(\mu) > 0$, no informative equilibrium exists.*

The following result formalizes the Blackwell paradox. While Theorem 2 is stated for binary signals, the paradox extends immediately to continuous signal structures via Blackwell dominance.

Proposition 5 (Blackwell Paradox under Endogenous Pricing). *Let G_B be the posterior distribution under the binary signal with signal precision $\mu > 4K + \kappa$ with $M(\mu) > 0$ used in Theorem 2, and G_C be a continuous posterior distribution with support $[1 - \mu, \mu]$ that is Blackwell-dominated by G_B . Then, although G_B is strictly more informative than G_C in the Blackwell sense, the equilibrium protest under endogenous pricing can be less informative:*

$$0 = \lambda_{G_B}^*(p^*(G_B)) \leq \lambda_{G_C}^*(p^*(G_C)),$$

where $\lambda_G^*(p^*(G))$ denotes the highest equilibrium informativeness under signal distribution G .

Proposition 5 is distinctive to settings with endogenous pricing. In models of protests against governments (Ekmekci and Lauermann, 2022), individual signal precision unambiguously improves protest informativeness, because there is no price to screen the protesting population.

4.3. Welfare Analysis

We examine social welfare, which is the sum of the firm's profit and consumer surplus. Then, social welfare is $((v - \kappa) \cdot \text{purchase} - K) \chi(\text{on sale})$. In an informative equilibrium with sufficiently small σ , the protest approximately perfectly reveals the true state v regardless of the size of μ . Therefore, social welfare is $(1 - \kappa - K)/2$. By contrast, in an uninformative equilibrium, social welfare is at most $(\mu - \kappa)/2 - K$. Therefore, the informative equilibrium attains a higher social welfare.

Our findings suggest that as consumers' private signals become more precise, social welfare can decrease as the informativeness λ vanishes. The following corollary is a summary.

Corollary 1. *Suppose that $\kappa + K > 1/2$. When σ is sufficiently small and w is large enough, social welfare is larger for $\mu \in (K + \kappa, 2K + \kappa)$ than that for $\mu > 4K + \kappa$ with $M(\mu) > 0$.*

5. Uncommitted price

5.1. Model

In the previous section, the firm committed to a price before observing the public signal. Here, we allow it to set the price ex post. The Blackwell paradox persists in this setting, but the mechanism differs. Under committed pricing, the information-rent effect drives the paradox: the firm's price compresses buyers' surplus, narrowing the gap in protest incentives between buyers and non-buyers and thereby reducing protest informativeness. Under uncommitted pricing, an additional force emerges: consumers strategically participate in protests to manipulate the firm's subsequent pricing. Higher turnout signals lower product value, inducing price reductions. This price-manipulation motive is stronger for high-signal consumers, who benefit more from lower prices because they are more likely to purchase. When signal precision is high, this motive reverses the natural participation pattern, high-signal consumers protest more than low-signal ones, rendering protests uninformative. Thus, under both pricing regimes, precise individual signals undermine protest informativeness, though the dominant mechanism differs.

The new decision flow is as follows:

1. Each consumer observes a private signal θ_i and decides whether to participate in the protest.
2. Turnout τ is realized, and the firm and consumers observe public signal $t = \tau + \varepsilon$.
3. The firm decides whether to continue selling the product, and if so, sets a price.
4. If the product is on sale, each consumer decides whether to buy it.

A crucial difference from the committed-price case is that turnout now influences the price: informative protests push the firm toward the low-price branch.¹⁷

5.2. Formal analysis of the uncommitted price case

To focus the discussion on whether an informative equilibrium exists, we assume that $\lambda_0 > \lambda_1$. Then we verify the consumers' incentives to participate in the protests. Note that even in the uncommitted price case, after the price is set, the consumers' decision criterion for purchasing the product is unchanged. Thus, a type- θ consumer purchases the product if and only if $t < T_\theta(p)$.

Given this strategy, the firm's profit is

$$\Pi(p, t) = \begin{cases} -K & \text{if } t > T_H(p) \\ [\Pr(v = 1 | t) \cdot \mu + \Pr(v = 0 | t) \cdot (1 - \mu)] \cdot (p - \kappa) - K & \text{if } t \in (T_L(p), T_H(p)] \\ p - \kappa - K & \text{if } t \leq T_L(p). \end{cases}$$

This implies that the optimal price is either $p = P_H(t)$ or $P_L(t)$. Therefore, we need to compare the following expressions.

$$\Pi_H^*(t) := [\Pr(v = 1 | t) \cdot (2\mu - 1) + 1 - \mu] \cdot (P_H(t) - \kappa) - K$$

$$\Pi_L^*(t) := P_L(t) - \kappa - K.$$

$\Pi_H^*(t)$ represents the profit when the firm focuses solely on type- H consumers, while $\Pi_L^*(t)$ indicates the profit when targeting both consumer types. Also, the firm discontinues the product sales rather than selling it when $\max\{\Pi_H^*, \Pi_L^*\} < 0$.

To simplify the discussion, we assume $\kappa = 0$ hereafter. The following lemma identifies the condition under which one profit exceeds the other.

Lemma 4. *Assume that $\kappa = 0$. Then, there exist values μ^* and $\mu^\dagger$ with $\mu^* < \mu^\dagger$ such that:*
(a) If $\mu > \mu^\dagger$, there is $\hat{t} < T_L(K)$ such that $\Pi_H^(t) < \Pi_L^*(t)$ if and only if $t < \hat{t}$. (b) If $\mu < \mu^\dagger$, $\Pi_H^*(t) < \Pi_L^*(t)$ for each $t < T_L(K)$. (c) $T_L(K) \rightarrow \infty$ if $\mu < \mu^*$, and $T_L(K) \rightarrow -\infty$ if $\mu > \mu^*$.*

In words, Lemma 4 shows that when μ is high, the firm sets $p = P_L$ for low t , $p = P_H$ for medium t , and discontinues for high t . If μ is intermediate, $p = P_L$ is optimal for low t and discontinuation is optimal for high t .

¹⁷ Indeed, protests tend to decrease prices. Hendel et al. (2017) empirically demonstrated that prices declined after a consumer boycott organized in Israel on Facebook.

Given the firm's behavior, we next consider consumers' incentives to participate in the protest. When μ is high, type- H consumers still buy near the firm's cutoff, so they have stronger incentives for price manipulation than type- L consumers, reversing the participation pattern and rendering protests uninformative. When μ is low, both types purchase while the product is sold, preserving $c_L > c_H$ and keeping protests informative. The next theorem formalizes these two regimes of μ .

Theorem 3. *Assume that $\lambda_0 > \lambda_1$.*

- (a) *Suppose that $\mu > \max\{\mu^\dagger, 2K\}$. Then, for sufficiently large σ , $c_H > c_L$ and for sufficiently small σ , $c_L > c_H$.*
- (b) *Suppose that $\mu \in (\mu^*, \mu^\dagger)$. Then, for sufficiently large σ , $c_L > c_H$.*

The inequality $c_H > c_L$ obtained in part (a) contradicts the maintained assumption $\lambda_0 > \lambda_1$, which requires $c_L > c_H$. Hence, the equilibrium must be uninformative.

Hence, protests become uninformative when μ is high and remain informative for intermediate μ for large w , mirroring the pattern under committed pricing. The Blackwell paradox thus persists: more precise individual signals can reduce collective informativeness regardless of whether pricing is committed or uncommitted.

Lastly, note that the price-manipulation incentive disappears when the noise level σ is small. In this case, even a marginal increase in participation sends a sufficiently strong signal that $v = 0$, prompting the firm to discontinue the product immediately and thus eliminating any incentive to manipulate prices.

6. Conclusion and Discussion

6.1. Summary and Implications

This paper identifies a new channel through which market power distorts information aggregation: a firm's pricing strategy screens its customer base, compressing the heterogeneity in protest incentives that is necessary for information transmission. The resulting Blackwell paradox under endogenous pricing, where more precise individual signals lead to less informative collective outcomes, is distinctive to markets with price-setting firms.

The effect of price on protest informativeness decomposes into a pivotality effect and an information-rent effect. In low-noise environments, the information-rent effect dominates, creating a price-informativeness trade-off. Under binary signals, this trade-off implies that higher signal precision leads to uninformative equilibria in which the firm extracts surplus at the cost of collective information.

Policy implications: Our results point to two complementary policy instruments. First, modest price-cap policies that keep prices inside the informative window (below $\bar{p}$) can preserve the disciplinary role of consumer activism. Second, improving the accuracy and transparency of protest reporting, for example through verified participation counts or standardized reporting protocols, can serve as a substitute for direct price regulation: precise public signals suppress the price-manipulation motive (Theorem 3) and restore the pivotal-information effect under uncommitted pricing.

Empirical predictions: The model generates several testable predictions. Consumer protests should be less effective at changing firm behavior for high-markup products, because the firm's price screens the customer base more aggressively, reducing protest informativeness. Relatedly, the composition of protest participants should be more homogeneous for high-priced products, skewed toward high-willingness-to-pay consumers. This can be tested using survey data on boycott participants. The model also predicts that consumer activism is more effective at transmitting information in industries with price regulation (e.g., utilities, pharmaceuticals), where regulatory constraints prevent price-based screening. Finally, online reviews for high-priced products should exhibit positive selection bias, consistent with empirical findings on review bias.

Broader applicability: While our formal analysis focuses on consumer protests, the core mechanism extends to other settings where a price determines the composition of a feedback pool. The mechanism is most directly applicable to online product reviews, where a high price compresses each buyer's surplus from the product, narrowing the gap in review incentives between satisfied and dissatisfied buyers and thereby reducing the informativeness of aggregate ratings. This shares a common structure with the self-selection bias documented by Li and Hitt (2008), who show that early buyers hold systematically different preferences from later buyers, generating biased review dynamics over time. Acemoglu et al. (2022) also develop a model of online reviews as social learning and identify the selection effect, although the price is exogenous. Similar logic applies to shareholder activism, where trading before proxy votes endogenously shifts the voter base, with the share price acting as a screening threshold (Levit et al., 2024). While their focus is on preference aggregation, the common structure—a price determines who participates in a collective decision—reinforces the generality of the screening mechanism. Formalizing these applications would require adapting the model; for instance, reviews are public rather than aggregated into a noisy signal, but the core trade-off between screening and informativeness is likely to persist.

6.2. Discussion

We now discuss our modeling assumptions.

Rule-utilitarian assumption: We assumed that consumers follow a rule-utilitarian principle when deciding whether to protest. This framework, standard in the literature on collective action in large economies (Feddersen and Sandroni, 2006; Ekmekci and Lauermann, 2022), sidesteps the well-known difficulty that each individual’s impact vanishes in a continuum economy. Kaufmann et al. (2024) address the same challenge through a different route, specifying preferences in which consumers directly value aggregate outcomes. Our results do not depend on this distinction: the main driver is the price–informativeness trade-off, which is shaped by the firm’s screening of the customer base, not by the specific form of consumers’ ethical motivation.

When each consumer maximizes overall consumer welfare rather than type-specific welfare, the participation cutoffs become $c_H^* = \mu c_H + (1 - \mu)c_L$ and $c_L^* = (1 - \mu)c_H + \mu c_L$. As long as $c_L > c_H$, type- L consumers remain more likely to participate, and the same conclusions follow.

Objective of participation in protest: We assumed that consumers consider their type’s aggregate utility when deciding whether to participate in the protest. However, as in Ali and Lin (2013), if this motivation stems from a reputation concern, it is more plausible to assume that consumers consider the expected overall consumer welfare. As shown above, this alternative does not change the qualitative results.

Modeling of a protest: Our model focuses on protest as an expressive act, distinct from the purchase decision itself (a boycott). This focus is motivated by the many contemporary consumer campaigns where public expression on platforms like social media is a central feature. Our contribution is to show how information aggregation can fail at this expressive stage. A richer model could integrate both channels—expression and purchasing—to study their dynamic interplay.

Our model allows protest costs c_i to be negative for some consumers, which admits a natural interpretation in terms of expressive utility. A consumer with $c_i < 0$ derives a direct benefit from participating in the protest—for instance, from publicly expressing opposition to a firm’s perceived wrongdoing (cf. Hamlin and Jennings, 2011; Morgan and Vardy, 2012, on expressive voting). Morgan and Vardy (2012) show that when voters derive expressive utility from choosing the “right” option, large committees can make worse decisions than small ones, because expressive motives crowd out instrumental (pivotal) incentives. Our model identifies a different threat to information aggregation: it is not expressive motives per se, but the firm’s pricing strategy that distorts the composition of the deliberating group, undermining the informativeness of aggregate turnout even when expressive participation is prevalent.

Mechanism design by the firm: In our model, the firm's pricing strategy is limited to setting a price. While a firm can indeed use its price-posting mechanism to elicit information, our analysis reveals that the two information mechanisms—protest and pricing—are in direct conflict. The firm's profit-maximizing use of its pricing power to screen consumers is precisely what destroys the informational value of the protest.

Furthermore, there is a fundamental limitation to using a price-posting mechanism for information elicitation related to the timing of information. In our model, the firm must incur a cost K to continue selling the product. Therefore, any strategy to learn from realized demand requires the firm to take a risk for this cost. If the product value turns out to be low ($v = 0$), the information arrives “too late,” only after the cost K has been sunk. The protest mechanism provides a signal before this irreversible cost is paid, highlighting its unique economic importance as a pre-commitment signaling device.

Appendix

A. Proofs

A.1. Proofs in Section 3

Proof of Theorem 1. We first show the following claim, which summarizes the limits of pivotal probability, ξ_θ .

Claim 1.

$$\lim_{\sigma \rightarrow 0} \xi_\theta(T_f(p)) = 0 \quad \text{and} \quad \lim_{\sigma \rightarrow 0} \frac{\xi'_\theta(T_f(p))}{\xi_\theta(T_f(p))} T'_f(p) = -\frac{1}{2} \frac{\theta - (1 - \theta)a^*(p)}{\theta + (1 - \theta)a^*(p)} \eta_a(p).$$

Proof of Claim 1. The first limit follows from equations $(T_f(p) - \lambda_1)/\sigma = \lambda(2\sigma)^{-1} + \sigma\lambda^{-1} \ln(a^*(p))$, and $\xi_\theta(T_f(p)) = \sigma^{-1} \varphi((T_f(p) - \lambda_1)/\sigma)[\theta + (1 - \theta)a]$, which implies $\lim_{\sigma \rightarrow 0} \xi_\theta(T_f(p)) = 0$.

The second limit follows from equations $T'_f(p) = \sigma^2 \lambda^{-1} \eta_a(p)$ and

$$\frac{\xi'_\theta(T_f(p))}{\xi_\theta(T_f(p))} = -\frac{\theta \frac{T_f(p) - \lambda_1}{\sigma^2} + (1 - \theta) \frac{T_f(p) - \lambda_0}{\sigma^2} a}{\theta + (1 - \theta)a}.$$

Since $T_f(p) - \lambda_1 \rightarrow \lambda/2$ and $T_f(p) - \lambda_0 \rightarrow -\lambda/2$ as $\sigma \rightarrow 0$, we conclude the second limit. ■

The next claim shows convergence rates of the derivatives of c_θ .

Claim 2. (1) $|\partial c_\theta(p)/\partial p| \propto |\xi_\theta(T_f(p))|$ if $\theta = \theta(p)$, and (2) for each $\theta > \theta(p)$, (2-1) $|\partial c_\theta(p)/\partial p| \propto |\xi'_\theta(T_f(p))T'_f(p)|$, (2-2) $|\partial c_\theta(p)/\partial \theta| \propto |\xi_\theta(T_f(p))|$, and (2-3) $|\partial^2 c_\theta(p)/\partial \theta \partial p| \propto |\xi'_\theta(T_f(p))T'_f(p)|$.

Proof of Claim 2. We have two cases. First, suppose that $P_\theta(T_f(p)) > p$. Then, we have

$$\frac{\partial c_\theta(p)}{\partial p} = \xi_\theta(T_f(p))w(1 - P'_\theta(T_f(p))T'_f(p)) - w[P_\theta(T_f(p)) - p]_+ \xi'_\theta(T_f(p))T'_f(p)$$

$$P'_\theta(T_f(p))T'_f(p) = -P_\theta(1 - P_\theta)\eta_a(p) < 0$$

Then, $|\partial c_\theta(p)/\partial p| \propto |\xi'_\theta(T_f(p))T'_f(p)|$ if $\theta > \theta(p)$ and $\lim_{\theta \nearrow \theta(p)} |\partial c_\theta(p)/\partial p| \propto |\xi_\theta(T_f(p))|$.

Next, $\partial c_\theta/\partial \theta$ is calculated as

$$\frac{\partial c_\theta}{\partial \theta} = -\xi_\theta(T_f(p))w \left[\frac{\theta}{[\theta + (1 - \theta)a^*(p)]^2} - [P_\theta(T_f(p)) - p]_+ \frac{\varphi\left(\frac{T_f(p) - \lambda_1}{\sigma}\right) - \varphi\left(\frac{T_f(p) - \lambda_0}{\sigma}\right)}{\sigma \xi_\theta(T_f(p))} \right].$$

Note that

$$\frac{1}{\sigma} \frac{\varphi\left(\frac{T_f(p) - \lambda_1}{\sigma}\right) - \varphi\left(\frac{T_f(p) - \lambda_0}{\sigma}\right)}{\xi_\theta(T_f(p))} = \frac{1 - a^*(p)}{\theta + (1 - \theta)a^*(p)},$$

which is independent of σ . Therefore, $|\partial c_\theta/\partial \theta| \propto \xi_\theta(T_f(p))$.

Finally, the cross derivative is calculated as

$$\frac{\partial^2 c_\theta(p)}{\partial \theta \partial p} = -\xi'_\theta(T_f(p))T'_f(p)Y(p) - \xi_\theta(T_f(p))Y'(p),$$

$$Y(p) = \frac{wa^*(p)}{\theta + (1 - \theta)a^*(p)} \left[\frac{\theta}{\theta + (1 - \theta)a^*(p)} + \frac{p(1 - a)}{a} \right].$$

As $Y(p)$ is independent of σ , $|\partial^2 c_\theta(p)/\partial \theta \partial p| \propto |\xi'_\theta(T_f(p))T'_f(p)|$. ■

To complete the proof, note that for each θ with $P_\theta > p$,

$$v_\theta := \frac{\sigma}{w\varphi\left(\frac{T_f(p) - \lambda_1}{\sigma}\right)} \frac{\partial c_\theta}{\partial p} = (\theta + (1 - \theta)a^*(p)) \left[1 + \left[P_\theta\left(\frac{1}{2} - p\right) + \frac{p}{2} \right] \eta_a(p) \right]$$

$$\frac{\partial v_\theta}{\partial \theta} = (1 - a^*(p)) \left[1 + \left[\frac{p - \frac{1}{2}}{a^*(p) - 1} + \frac{p}{2} \right] \eta_a(p) \right].$$

Then,

$$\frac{\partial^2 c_\theta}{\partial p \partial \theta} > 0 \text{ if and only if } (1 - a^*(p)) \left[1 + \left[\frac{p - \frac{1}{2}}{a^*(p) - 1} + \frac{p}{2} \right] \eta_a(p) \right] > 0. \quad (1)$$

On the other hand, if $\theta = \theta(p)$, as $P_\theta = p$, $\partial c_\theta(p)/\partial p > 0$ if and only if $1 + p(1 - p)\eta_a(p) > 0$. This value is positive, as the following claim shows.

Claim 3 (Elasticity bound). *If $\partial a^*(p)/\partial p < 0$,*

$$-\frac{1}{(1-p)p} < -\frac{1}{2} \frac{2p-1}{(1-p)p} < \eta_a(p)$$

Proof of Claim 3. Recall that $a^*(p)$ is the solution to

$$\frac{K}{p - \kappa} = \frac{1}{1 + a} [1 - \tilde{G}_1(\omega(p, a))] + \frac{a}{1 + a} [1 - \tilde{G}_0(\omega(p, a))]$$

with respect to a . If $\partial a^*(p)/\partial p < 0$, $\omega(p, a) = 2 \ln(a^*(p)) + \ln(p/(1 - p))$ should increase in p . Otherwise, the right-hand side increases while the left-hand side decreases, a contradiction. Then, $\partial \omega(p, a)/\partial p > 0$ if and only if

$$-\frac{1}{2} \frac{2p-1}{(1-p)p} < \eta_a(p)$$

The last inequality follows from $p \leq 1$. ■

Recall that $\xi_\theta(T_f(p)) \rightarrow 0$ as $\sigma \rightarrow 0$ by Claim 1. Note that by a calculation,

$$\begin{aligned} \frac{\partial \Lambda(\lambda, p)}{\partial p} &= -f(c_{\theta(p)}(p)) \frac{\partial c_{\theta(p)}(p)}{\partial p} (G_0(\theta(p)) - G_1(\theta(p))) \\ &\quad - \int_{\theta(p)} \left[f(c_\theta(p)) \frac{\partial^2 c_\theta(p)}{\partial \theta \partial p} + f'(c_\theta(p)) \frac{\partial c_\theta(p)}{\partial \theta} \frac{\partial c_\theta(p)}{\partial p} \right] (G_0(\theta) - G_1(\theta)) d\theta. \end{aligned}$$

By using Claim 2, for each $\theta > \theta(p)$, we have

$$\frac{\frac{\partial c_\theta(p)}{\partial p} \frac{\partial c_\theta(p)}{\partial \theta}}{\frac{\partial^2 c_\theta(p)}{\partial \theta \partial p}} \rightarrow 0 \text{ as } \sigma \rightarrow 0.$$

Therefore, $\partial \Lambda(\lambda, p)/\partial p < 0$ if

$$\frac{\partial^2 c_\theta(p)}{\partial \theta \partial p} > 0 \text{ for } \theta > \theta(p) \text{ and } \frac{\partial c_{\theta(p)}(p)}{\partial p} > 0.$$

The latter inequality always holds by Claim 3. Thus, by equation (1), we have that $\partial \Lambda(\lambda, p)/\partial p < 0$ if

$$(1 - a^*(p)) \left[1 + \left[\frac{p - \frac{1}{2}}{a^*(p) - 1} + \frac{p}{2} \right] \eta_a(p) \right] > 0.$$

■

Proof of Lemma 2. Note that the profit is written as

$$\Pi_\lambda(p) = \frac{\lambda}{2\sigma^3} \int_0^{a^*(p)} \Pi^*(p, a) \left[\varphi\left(\frac{\sigma}{\lambda} \ln(a) + \frac{\lambda}{2\sigma}\right) + \varphi\left(\frac{\sigma}{\lambda} \ln(a) - \frac{\lambda}{2\sigma}\right) \right] a \, da.$$

and

$$\varphi\left(\frac{\sigma}{\lambda} \ln(a) + \frac{\lambda}{2\sigma}\right) = \exp\left(-\frac{1}{2}\left(\frac{\sigma}{\lambda}\right)^2 (\ln a)^2 - \frac{1}{8}\left(\frac{\lambda}{\sigma}\right)^2\right) a^{-\frac{1}{2}}$$

Then, by letting $\rho = \lambda/\sigma$ and $W_\rho(a) = \exp(-\rho^{-2}(\ln a)^2/2 - \rho^2/8)[a^{1/2} + a^{3/2}]$, the expected profit is written as $\Pi_\lambda(p) = \lambda(2\sigma^3)^{-1} \int_0^{a^*(p)} \Pi^*(p, a) W_\rho(a) \, da$.

The following claim shows that at the limit $\sigma \rightarrow 0$ or $\sigma \rightarrow \infty$ (equivalently, $\rho \rightarrow \infty$ or $\rho \rightarrow 0$), $W_\rho(\exp(\rho))$ dominates other $W_\rho(a)$.

Claim 4. For any $a \in \mathbb{R}_+$, $W_\rho(a)/W_\rho(\exp(\rho)) \rightarrow 0$ as $\rho \rightarrow 0$ or $\rho \rightarrow \infty$.

Proof of Claim 4. By a calculation,

$$\frac{W_\rho(a)}{W_\rho(\exp(\rho))} = \exp\left(-\rho^{-2} \frac{(\ln(a))^2}{2}\right) \frac{\left[a^{\frac{1}{2}} + a^{\frac{3}{2}}\right]}{[\exp(\rho)]^{\frac{1}{2}} + [\exp(\rho)]^{\frac{3}{2}}}$$

This converges to 0 if either $\sigma \rightarrow 0$ or $\sigma \rightarrow \infty$. ■

Therefore, when $\sigma \rightarrow 0$ or $\sigma \rightarrow \infty$, $\Pi_\lambda(p) \rightarrow \lambda(2\sigma^3)^{-1} \Pi^*(p, \exp(\rho))$, which implies that $p_\sigma^* \rightarrow p_{a_\sigma^\dagger}$. ■

Proof of Proposition 1. Before the proof, we prepare the following claim:

Claim 5. Suppose that the assumptions of Proposition 1 hold. Then, we have

1. When $\sigma \rightarrow 0$, $p_\sigma^\dagger \rightarrow 1$.
2. $\lim_{\sigma \rightarrow 0} \omega(p_\sigma^\dagger, a^*(p_\sigma^\dagger)) < \infty$ and $a^*(p_\sigma^\dagger) \rightarrow 0$.
3. $\lim_{\sigma \rightarrow 0} \eta_a(p_\sigma^\dagger)(1 - p_\sigma^\dagger)p_\sigma^\dagger = -1/2$.

Proof of Claim 5. Note that

$$\begin{aligned} \frac{\partial \Pi^*(p, a)}{\partial p} &= \sum \pi_v \left(1 - \tilde{G}_v(\omega(p, a)) - (p - \kappa) \frac{\tilde{g}_v(\omega(p, a))}{p(1-p)} \right) \\ \frac{\partial \Pi^*(p, a)}{\partial a} &= \frac{1}{a} \left[\pi_0 \pi_1 (\tilde{G}_1(\omega(p, a)) - \tilde{G}_0(\omega(p, a))) - 2(p - \kappa) \sum_v \pi_v \tilde{g}_v(\omega(p, a)) \right], \end{aligned}$$

where, $\pi_1 = 1/(1+a)$, and $\pi_0 = a/(1+a)$.

Note that the optimal price should be larger than $K > 0$. Suppose by contradiction that $\lim_{\sigma \rightarrow 0} p_\sigma^\dagger < 1$. Then, as $\omega(p, a) = 2 \ln(a) + \ln(p/(1-p))$ and $a_\sigma^\dagger \rightarrow \infty$, $\omega(p, a) \rightarrow \infty$. Then, by the assumption that $\lim_{x \rightarrow \infty} (1 - \tilde{G}_v(x))/(\tilde{g}_v(x)) = \infty$, $\partial \Pi^*(p, a)/\partial p > 0$ for any p . This implies that $p_\sigma^\dagger \rightarrow 1$.

Next, consider $\omega(p_\sigma^\dagger, a^*(p_\sigma^\dagger))$. If $\omega(p_\sigma^\dagger, a^*(p_\sigma^\dagger)) \rightarrow \infty$, $\Pi(p_\sigma^\dagger, a^*(p_\sigma^\dagger)) \rightarrow 1 - K > 0$, which is a contradiction. Furthermore, as $\lim_{\sigma \rightarrow 0} \omega(p_\sigma^\dagger, a^*(p_\sigma^\dagger)) < \infty$ and $p_\sigma^\dagger \rightarrow 1$, $a^*(p_\sigma^\dagger) \rightarrow 0$.

Finally, consider the elasticity of $a^*(p)$, which is calculated by

$$\begin{aligned} \eta_a(p) &= \frac{\frac{\partial a^*(p)}{\partial p}}{a^*(p)} = -\frac{1}{a} \frac{\frac{\partial \Pi^*(p, a)}{\partial p}}{\frac{\partial \Pi^*(p, a)}{\partial a}} \\ &= -\frac{\sum \pi_v \left(1 - \tilde{G}_v(\omega(p, a)) - (p - \kappa) \frac{\tilde{g}_v(\omega(p, a))}{p(1-p)} \right)}{\pi_0 \pi_1 (\tilde{G}_1(\omega(p, a)) - \tilde{G}_0(\omega(p, a))) - 2 \sum_v \pi_v (p - \kappa) \tilde{g}_v(\omega(p, a))}. \end{aligned}$$

Note that $\pi_0 \rightarrow 0$ as $a^*(p_\sigma^\dagger) \rightarrow 0$. Then, as $\lim_{\sigma \rightarrow 0} \omega(p_\sigma^\dagger, a^*(p_\sigma^\dagger)) < \infty$,

$$\begin{aligned} \eta_a(p)p(1-p) &= -\frac{\sum \pi_v [p(1-p)(1 - \tilde{G}_v(\omega(p, a))) - (p - \kappa) \tilde{g}_v(\omega(p, a))]}{\pi_0 \pi_1 (\tilde{G}_1(\omega(p, a)) - (p - \kappa) \tilde{G}_0(\omega(p, a))) - 2(p - \kappa) \sum_v \pi_v \tilde{g}_v(\omega(p, a))} \\ &\rightarrow -\frac{-(p - \kappa) \sum \pi_v \tilde{g}_v(\omega(p, a))}{-2(p - \kappa) \sum_v \pi_v \tilde{g}_v(\omega(p, a))} \quad (\text{since } p \rightarrow 1 \text{ and } \pi_0 \rightarrow 0). \end{aligned}$$

Therefore, $\lim_{\sigma \rightarrow 0} p_\sigma^\dagger (1 - p_\sigma^\dagger) \eta_a(p_\sigma^\dagger) = -1/2$. ■

Now we complete the proof. We use the sufficient condition shown in Theorem 1. When $a^*(p) < 1$ and $\partial a^*(p)/\partial p < 0$,

$$\begin{aligned} (1 - a^*(p)) \left[1 + \left[\frac{p - \frac{1}{2}}{a^*(p) - 1} + \frac{p}{2} \right] \eta_a(p) \right] &= 1 - a^*(p) + \left[\frac{1}{2}(1 - p) - \frac{1}{2} p a^*(p) \right] \eta_a(p) \\ &> 1 - a^*(p) + \frac{1}{2}(1 - p) \eta_a(p). \end{aligned}$$

As $\lim_{\sigma \rightarrow 0} \eta_a(p_\sigma^\dagger)(1 - p_\sigma^\dagger) = -1/2$, and $a^*(p_\sigma^\dagger) \rightarrow 0$ as $\sigma \rightarrow 0$, at $p = p_\sigma^\dagger$

$$\lim_{\sigma \rightarrow 0} \left[1 - a^*(p) + \frac{1}{2}(1 - p) \eta_a(p) \right] = \frac{3}{4} > 0.$$

Then, by Theorem 1, $\lim_{\sigma \rightarrow 0} \partial \Lambda(\lambda, p)/\partial p|_{p=p_\sigma^\dagger} < 0$. ■

A.2. Proofs in Section 4

Proof of Lemma 3. To simplify the notation, in this proof, we use T as the firm's cutoff. Note that the cutoff depends on the consumers' behavior. Therefore, we consider the cases (i) $t \geq T_H(p)$, (ii) $t \in (T_L(p), T_H(p))$, and (iii) $t \leq T_L(p)$.

(i) Suppose that $t \geq T_H(p)$. Note that if $t > T_H(p)$, the firm discontinues the sale as no consumer purchases the product, which leads to a deficit. Consider the case $t = T_H(p)$. Then, as only type- H consumers purchase the product, the firm's expected profit when the public signal t is observed is $\Pi_H(t)$, which is defined by

$$\Pi_H(t) := w \cdot ([\Pr(v = 1 | t) \cdot \mu + \Pr(v = 0 | t) \cdot (1 - \mu)] \cdot (p - \kappa) - K).$$

The condition when $\Pi_H(t) > 0$ is summarized as follows.

Claim 6. (1) If $p - \kappa > K/(1 - \mu)$, $\Pi_H(t) > 0$. (2) If $K/\mu > p - \kappa$, $\Pi_H(t) < 0$. (3) If $K/(1 - \mu) > p - \kappa > K/\mu$, $\Pi_H(t) > 0$ if and only if $t < \tilde{T}$, where

$$\tilde{T} := \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)}{\lambda_0 - \lambda_1}.$$

Using this claim, $T = T_H(p)$ if either (a) $p - \kappa > K/(1 - \mu)$, or (b) $K/(1 - \mu) > p - \kappa > K/\mu$ and $\tilde{T} \geq T_H(p)$. Otherwise, at $t = T_H(p)$, the profit is negative, and therefore, the cutoff is below $T_H(p)$.

(ii) Suppose that $T_L(p) < t < T_H(p)$. Then, at $t = T$, only the type- L consumer purchases the product. In this case, the firm's profit is $\Pi_H(t)$. Therefore, if $K/(1 - \mu) > p - \kappa > K/\mu$ and $\tilde{T} \in (T_L(p), T_H(p))$, $T = \tilde{T}$. Otherwise, there is no t such that $\Pi_H(t) = 0$.

(iii) and (iv) Suppose that $t \leq T_L(p)$. Then, the firm's profit is $P_L := w[(p - \kappa) - K]$. Therefore, $P_L \geq 0$ if and only if $p - \kappa \geq K$.

The remaining cases to consider the cutoff point are the following.

(a) $p - \kappa \geq K/\mu$ and $T_L(p) \geq \tilde{T}$. Then, the profit is positive if and only if $t < T_L(p)$, the cutoff is $T_L(p)$.

(b) $K/\mu > p - \kappa \geq K$. Then, the profit is positive if and only if $t < T_L(p)$, and the cutoff is also $T_L(p)$.

(c) $K > p - \kappa$. Then, the profit is negative for any t . Therefore, the firm discontinues the sale; in other words, the cutoff is $-\infty$. ■

A.2.1. Proof of Proposition 2

Before the proof of Proposition 2, we prepare the following claim.

Claim 7. Define $\underline{p}$ to be the largest solution to the following equation with respect to p :

$$\frac{1-\mu}{\mu} \frac{1-p}{p} = \frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}.$$

Then, $T_f(p) = T_H(p)$ if and only if $p > \bar{p}$ and $T_f(p) > T_L(p)$ if and only if $p > \underline{p}$. Further, $K/(1-\mu) + \kappa > \bar{p} > \underline{p} > K/\mu + \kappa$.

Proof of Claim 7. (a) If $p \geq K/(1-\mu) + \kappa$, $T_f(p) = T_H(p)$ by Lemma 3. If $p < K/(1-\mu) + \kappa$, $\tilde{T} < T_H(p)$ if and only if

$$\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)} < \frac{\mu}{1-\mu} \frac{1-p}{p},$$

which is equivalently $p > \bar{p}$. Further, a simple calculation shows $K/(1-\mu) + \kappa > \bar{p}$.

(b) If $p < K/\mu + \kappa$, by Lemma 3, $T_f(p) = -\infty$. Consider the case where $p \geq K/\mu + \kappa$. Then, $T_f(p) = \max\{T_L(p), \tilde{P}\}$. Note that $T_L(p) < \tilde{T}$ if and only if

$$\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)} > \frac{1-\mu}{\mu} \frac{1-p}{p},$$

equivalently, $g(p) > 0$, where g is a convex quadratic function (illustrated in Figure 3). Note that $g(p) < 0$ at $p = K/\mu + \kappa$. Then, $T_L(p) < T_f(p)$ if and only if $p > \underline{p}$. ■

Proof of Proposition 2. (1) Suppose that $p \geq \bar{p}$. If $p \geq \bar{p}$, by Claim 7, $T_f(p) = T_H(p)$. This implies that $c_\theta = -w \cdot [P_\theta(T_f(p)) - p]_+ \cdot \xi_\theta(T_f(p)) = 0$ for each $\theta \in \{\theta_H, \theta_L\}$. Therefore, $\lambda = \lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H)) = 0$, which implies that there is no informative equilibrium.

(2) Consider the case $p \in (\underline{p}, \bar{p})$. Then, if $\lambda_0 > \lambda_1$, $T_L(p) < T_f(p) < T_H(p)$ for any c_H, c_L .

We can easily verify that $\Lambda(\lambda, p) := \lambda_0 - \lambda_1 = (2\mu - 1) \cdot (F(c_L) - F(c_H))$ is continuous.

Below, we show that $\Lambda(\lambda, p) > \lambda$ for some $\lambda > 0$ when w is large enough. By the definition of $\bar{p}$ and $\underline{p}$, $P_H(T_f(p)) > p$, and $p > P_L(T_f(p))$. This implies that

$$c_H = -w[P_H(T_f(p)) - p] \cdot \xi_H(T_f(p)), \text{ and } c_L = 0.$$

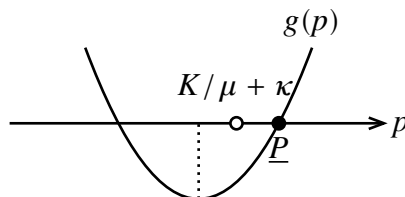


Figure 3: Illustration of g

Then,

$$c_L - c_H = w \cdot [P_H(T_f(p)) - p] \cdot \xi_H(T_f(p)) > 0.$$

Fix $\bar{\lambda}$ so that $0 < \bar{\lambda} < (2\mu - 1)F(0)$. Then,

$$(2\mu - 1)(F(c_L) - F(c_H)) > \bar{\lambda} \Leftrightarrow F^{-1}\left(F(0) - \frac{\bar{\lambda}}{2\mu - 1}\right) > c_H.$$

Now take w so that

$$\frac{-F^{-1}\left(F(0) - \frac{\bar{\lambda}}{2\mu - 1}\right)}{[P_H(T_f(p)) - p] \cdot \xi_H(T_f(p))} < w.$$

Then, $\Lambda(\bar{\lambda}, p) > \bar{\lambda}$. The intermediate value theorem implies the existence of a fixed point of Λ , which is in $(\bar{\lambda}, 2\mu - 1)$. This implies that the fixed point is a positive value and shows the existence of an informative equilibrium. Moreover, the equilibrium $\lambda^* \geq (2\mu - 1)F(0) - \delta$ for arbitrary $\delta \geq 0$ by taking $\bar{\lambda}$ close to $(2\mu - 1)F(0)$.

(3) Consider the case $p < \underline{p}$. Then, by Lemma 3 and Claim 7, if $p - \kappa > K$, $T_f(p) = T_L(p)$, and otherwise, $T_f(p) = -\infty$.

Suppose that $p - \kappa > K$. Then, by the definition of $T_L(p)$,

$$c_H = -w[P_H(T_L(p)) - p] \cdot \xi_H(T_f(p)), \text{ and } c_L = 0.$$

A similar way to case (2) shows the existence of an informative equilibrium.

If $p - \kappa \leq K$, as $T_f(p) = -\infty$. Then, the equilibrium becomes uninformative. ■

A.2.2. Proof of Proposition 3

Before we prove Proposition 3, we provide the following claim, which immediately follows from the definition of $\bar{p}$.

Claim 8. $\mu > \bar{p}$ if and only if $(\mu - \kappa)/2 > K$.

Now, we proceed to the proof of Proposition 3. Suppose that $(\mu - \kappa)/2 > \max\{K, 1 - \mu - \kappa\}$. This implies that $\mu > (2 - \kappa)/3$. Note that if $p = \mu$, as $(\mu - \kappa)/2 > K$, $p > \bar{p}$ by Claim 8. By Proposition 2, the equilibrium is uninformative and guarantees the profit $(\mu - \kappa)/2 - K$.

Below, we examine the existence of an informative equilibrium that yields a profit larger than $(\mu - \kappa)/2 - K$.

The expected profit is

$$\begin{aligned}\Pi(p) = & \sum_{v' \in \{0,1\}} \frac{1}{2} \left[\Phi\left(\frac{T_f(p) - \lambda_{v'}^*(p)}{\sigma}\right) \cdot [(v'\mu + (1-v')(1-\mu))(p - \kappa) - K] \right. \\ & \left. + \Phi\left(\frac{T_L(p) - \lambda_{v'}^*(p)}{\sigma}\right) \cdot ((1-v')\mu + v'(1-\mu))(p - \kappa) \right].\end{aligned}$$

Let $r = \lambda/(2\sigma)$. From Lemma 3, at the informative equilibrium, $T_L(p) \leq T_f(p) < T_H(p)$, and then, there are x_L, x_f such that for each $b \in \{L, H, f\}$, $(T_b(p) - \lambda_1)/\sigma = r + x_b$ and $(T_b(p) - \lambda_0)/\sigma = -r + x_b$, where $x_L < x_f < x_H$. Furthermore, as $p > \kappa + K > 1/2$ by Proposition 2,

$$x_L = \ln\left(\frac{1-\mu}{\mu} \frac{1-p}{p}\right) \frac{1}{2r} < x_f < x_H = \ln\left(\frac{\mu}{1-\mu} \frac{1-p}{p}\right) \frac{1}{2r} < -x_L.$$

Then, $x_L < -x_f$ implies that $\Phi(x_L + r) \leq \Phi(-x_f + r)$ and $\Phi(x_L - r) \leq \Phi(-x_f - r)$. By using this inequality and the symmetry of Φ ,

$$\begin{aligned}\Pi(p) & < \frac{\Phi(x_f + r) + \Phi(-x_f - r)}{2} \mu(p - \kappa) + \frac{\Phi(x_f - r) + \Phi(-x_f + r)}{2} (1 - \mu)(p - \kappa) \\ & \quad - \frac{K}{2} [\Phi(x_f + r) + \Phi(x_f - r)] \\ & = \frac{p - \kappa}{2} - \frac{K}{2} [\Phi(x_f + r) + \Phi(x_f - r)].\end{aligned}$$

Furthermore,

$$x_f = \ln\left(\max\left\{\frac{1-\mu}{\mu} \frac{1-p}{p}, \frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right\}\right)$$

If $p > \kappa + 2K$, $x_f > 0$. Then, $\Phi(x_f + r) + \Phi(x_f - r)$ is decreasing in r and thus, no less than 1. Therefore, $\Pi(p) < (p - \kappa - K)/2$. If $p < \kappa + 2K$, the maximum profit is K .

As $p < \bar{p}$, the maximum profit is

$$\frac{\bar{p} - \kappa - K}{2} = \frac{K}{2} \frac{\mu + \kappa(2\mu - 1)}{\mu(1 - \mu) + (2\mu - 1)K} - \frac{K}{2}.$$

Therefore, uninformative profit exceeds it if $\mu > 4K + \kappa$ and

$$\mu - \kappa > K \frac{\mu + \kappa(2\mu - 1)}{\mu(1 - \mu) + (2\mu - 1)K} + K.$$

A.2.3. Proof of Proposition 4

The proof consists of the following two claims, which hold under the assumption of Proposition 4.

Claim 9. *If the price induces an uninformative equilibrium, the expected profit is at most 0.*

Claim 10. By setting $p = \mu$, the equilibrium is informative. The expected profit is no less than $(\mu - \kappa - K)/2$ in this case.

Proof of Claim 9. By Proposition 2, the situation is either $p > \bar{p}$ or $p < \kappa + K$. From Claim 8 and assumption $(\mu - \kappa)/2 - K < 0, \bar{p} > \mu$. If $p > \bar{p} > \mu$, as the ex post belief of type H , $P_H = \mu < p$ at the uninformative equilibrium, the profit is 0 as the type- H never purchases. At the uninformative equilibrium with $p < \kappa + K$, the profit is also 0 as the firm gives up selling. ■

Proof of Claim 10. From Claim 8, $K > (\mu - \kappa)/2$ implies that $\bar{p} > \mu > 1 - \mu$, and as $\mu > K + \kappa$, Proposition 2 implies that an equilibrium exists and it is necessarily informative at $p = \mu$.

Recall that the expected profit is

$$\begin{aligned} \Pi(p) = \sum_{v' \in \{0,1\}} \frac{1}{2} & \left[\Phi\left(\frac{T_f(p) - \lambda_{v'}^*(p)}{\sigma}\right) \cdot [(v'\mu + (1-v')(1-\mu))(p - \kappa) - K] \right. \\ & \left. + \Phi\left(\frac{T_L(p) - \lambda_{v'}^*(p)}{\sigma}\right) \cdot ((1-v')\mu + v'(1-\mu))(p - \kappa) \right]. \end{aligned}$$

By Proposition 2, we can take λ as a positive value by taking w large. Then, by $\sigma \rightarrow 0$,

$$\Pi(\mu) \rightarrow \frac{\mu - \kappa - K}{2}.$$

■

Proof of Proposition 4. By Claims 9 and 10, and assumption $\mu > \kappa + K$, a price that leads to an informative equilibrium yields a positive profit, which is larger than the profit of any uninformative equilibrium. Then, the optimal price leads to an informative equilibrium. ■

A.3. Proofs in Section 5

Proof of Lemma 4. First note that for sufficiently small t , $\Pi_L^*(t) > \Pi_H^*(t)$. This is because for sufficiently small $t \approx -\infty$, $P_\theta(t) \approx 1$ and therefore $\Pi_L^*(t) \approx 1 > \mu \approx \Pi_H^*(t)$.

Recall that $P_\theta(t)$ is written as $P_\theta(t) = 1/(1 + a(1 - \theta)/\theta)$, where $a = \exp((\lambda_0 - \lambda_1)\sigma^{-2}(t - (\lambda_0 + \lambda_1)/2))$ and also $\Pr(v = 1 | t) = 1/(1 + a)$. To simplify the notation, let denote $P(t) = \Pr(v = 1 | t)$. By differentiation, if $\lambda_0 > \lambda_1$, we have

$$P'_\theta(t) = -\frac{\lambda_0 - \lambda_1}{\sigma^2} \cdot \frac{1 - \theta}{\theta} \cdot a \cdot (P_\theta(t))^2 < 0.$$

Note that we can write $P_H(t) = 1/(1 + a(1 - \mu)/\mu)$ and $P_L(t) = 1/(1 + a\mu/(1 - \mu))$. This implies that $P_H(t)/P_L(t)$ and $P(t)/P_L(t)$ are increasing in t .

Now we calculate the ratio of the profits:

$$\frac{\Pi_H^*(t)}{\Pi_L^*(t)} = (P(t)(2\mu - 1) + (1 - \mu)) \frac{P_H(t)}{P_L(t)} > 1 \Leftrightarrow Z(a) > 0, \quad (2)$$

$$Z(a) := (1 - \mu)\mu^2 a^2 - (2\mu - 1)(1 - (1 - \mu)\mu)a - \mu(1 - \mu)^2 > 0.$$

Note that $Z(a)$ is a convex quadratic function of a , and $Z(0) < 0$. Also note that as $P_L(t) > K$, $a < (1/K - 1) \cdot (1 - \mu)/\mu$. A calculation shows that $Z((1/K - 1) \cdot (1 - \mu)/\mu)$ is decreasing in μ , and let $\mu^\dagger$ be the solution that $Z((1/K - 1) \cdot (1 - \mu)/\mu) = 0$. This implies that if $\mu < \mu^\dagger$, $\Pi_H^*(t) < \Pi_L^*(t)$ for each t . On the contrary, if $\mu > \mu^\dagger$, there is $\hat{t} < T_L(K)$ such that $\Pi_H^*(t) < \Pi_L^*(t)$ if and only if $t < \hat{t}$.

(c) As $T_L(K)$ is the solution of $P_L(t) = K$, That is,

$$T_L(K) = \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{\lambda_0 - \lambda_1} \cdot \ln\left(\left(\frac{1}{K} - 1\right) \cdot \frac{1 - \mu}{\mu}\right).$$

Let μ^* be the solution to $(1/K - 1) \cdot (1 - \mu)/\mu = 1$. This exists if $K < 1/2$. Then, if $\mu < \mu^*$, $T_L(K) \rightarrow \infty$. In contrast, if $\mu > \mu^*$, $T_L(K) \rightarrow -\infty$. We can also verify that $Z((1/K - 1) \cdot (1 - \mu^*)/\mu^*) < 0$, that is, $Z(1) < 0$. This implies that $\mu^* < \mu^\dagger$. ■

A.3.1. Proof of Theorem 3

Proof of case (a): We first consider the high μ case: there is $\hat{t} < T_L(K)$ such that $\Pi_H^*(t) > \Pi_L^*(t)$ if and only if $t < \hat{t}$.

Let T^* be the solution to $\Pi_H^*(t) = 0$ with respect to t . This implies that at the optimal strategy, the firm sets price $P_L(t)$ when $t \in (-\infty, \hat{t})$, sets price $P_H(t)$ when $t \in (\hat{t}, T^*)$, and when $t > T^*$, discontinues the product sale.

Under this firm's behavior, the expected utility of the type- H is

$$w \left[\int_{\hat{t}-d\tau}^{T^*-d\tau} [P_H(t) - P_H(t + d\tau)] \xi_H(t) dt + \int_{\hat{t}-d\tau}^{\hat{t}-d\tau} [P_H(t) - P_L(t + d\tau)] \cdot \xi_H(t) dt \right].$$

Here, $P_H(t)$ is the expected value of the product for type- H while the price is set to $p = P_L(t)$. A marginal increase in turnout $d\tau$ affects the price. As the firm sets price $p = P_H(t)$ if $t > \hat{t}$, type- H consumers' informational rent $P_H(t) - P_L(t)$ vanishes. The differentiation concerning $d\tau$ is the marginal gain by participation. Therefore, type- H consumer i participates in the protest if and only if $c_H \geq c_i$, where

$$c_H := w \cdot \left[\underbrace{-[P_H(\hat{t}) - P_L(\hat{t})] \xi_H(\hat{t})}_{\text{information rent}} + \underbrace{\int_{\hat{t}}^{T^*} [-P_H'(t)] \xi_H(t) dt + \int_{\hat{t}}^{\hat{t}} [-P_L'(t)] \xi_H(t) dt}_{\text{benefit from price reduction by participation}} \right].$$

Now we compare with the type- L consumers' utility. The utility of the type- L is $w \cdot \int^{\hat{t}-d\tau} [P_L(t) - P_L(t+d\tau)] \cdot \xi_L(t) dt$ as the firm extracts type- L consumers' full surplus. Then, the type- L consumers participate in the protest if and only if

$$c_L := w \cdot \int^{\hat{t}} [-P'_L(t)] \xi_L(t) dt \geq c_i.$$

Then, $c_L > c_H$ if and only if

$$D := \int^{\hat{t}} [-P'_L(t)] (\xi_L(t) - \xi_H(t)) dt - \int_{\hat{t}}^{T^*} [-P'_H(t)] \xi_H(t) dt + [P_H(\hat{t}) - P_L(\hat{t})] \xi_H(\hat{t}),$$

which is shown to be negative.

Claim 11. Suppose that $\mu > \max\{\mu^\dagger, 2K\}$. Then, $D < 0$ for sufficiently large σ , and $D > 0$ for sufficiently small σ .

Proof of Claim 11. Recall that $a = \exp((\lambda_0 - \lambda_1)\sigma^{-2} \cdot [t - (\lambda_1 + \lambda_0)/2])$ and P_θ is written as $P_\theta = 1/(1 + a(1 - \theta)/\theta)$. Then,

$$\begin{aligned} P_H(t) - P_L(t) &= \frac{\frac{\mu}{1-\mu} - \frac{1-\mu}{\mu}}{\left(1 + \frac{\mu}{1-\mu}a\right) \cdot \left(1 + \frac{1-\mu}{\mu}a\right)} a, \\ -P'_\theta(t) &= \frac{\lambda_0 - \lambda_1}{\sigma^2} \frac{1 - \theta}{\theta} \cdot a \cdot (P_\theta(t))^2 > 0. \\ \xi_L(t) - \xi_H(t) &= \frac{2\mu - 1}{\sigma} \cdot \varphi\left(\frac{t - \lambda_1}{\sigma}\right) \cdot [a - 1]. \end{aligned}$$

As shown in the proof of Lemma 4, $\Pi_H^*(t) = \Pi_L^*(t)$ if and only if $Z(a) = 0$. (Z is defined in (2)). Therefore, when we write $\hat{a} = \exp((\lambda_0 - \lambda_1)\sigma^{-2}(\hat{t} - (\lambda_1 + \lambda_0)/2))$, $Z(\hat{a}) = 0$. Also, this implies that $P_H(\hat{t}) - P_L(\hat{t})$ is independent of σ .

We first consider a large σ case. Then, even $\sigma \rightarrow \infty$, $\hat{a}$ is a finite value, which implies that $\hat{t}/\sigma^2$ converges to a finite real number. Further,

$$\frac{\xi_H(t)}{\xi_H(\hat{t})} = \frac{\mu \cdot \exp\left((\hat{t} - t) \cdot \frac{\hat{t} + t - 2\lambda_1}{2\sigma^2}\right) + (1 - \mu) \cdot \exp\left((\hat{t} - t + \lambda_0 - \lambda_1) \cdot \frac{\hat{t} + t - (\lambda_0 + \lambda_1)}{2\sigma^2}\right)}{\mu + (1 - \mu) \cdot \hat{a}}.$$

As $\hat{t}/\sigma^2$ converges to a real number, for each t with $|t| < |\hat{t}| + z$ for some constant z , $\xi_H(t)/\xi_H(\hat{t}) \rightarrow \infty$. In contrast, if $t < \hat{t}$, $\xi_H(t)/\xi_H(\hat{t}) \rightarrow 0$ as $\hat{t} \rightarrow -\infty$. By Lemma 4, as $\mu > \mu^\dagger > \mu^*$, $T_L(K) \rightarrow -\infty$ and $\hat{t} < T_L(K)$. Then, $\hat{t} \rightarrow -\infty$ as $\sigma \rightarrow \infty$.

Dividing D by $\xi_H(\hat{t})$ yields that

$$\int^{\hat{t}} [-P'_L(t)] \frac{\xi_L(t) - \xi_H(t)}{\xi_H(\hat{t})} dt - \int_{\hat{t}}^{T^*} [-P'_H(t)] \frac{\xi_H(t)}{\xi_H(\hat{t})} dt + [P_H(\hat{t}) - P_L(\hat{t})]. \quad (3)$$

Note also that $\xi_L(t) < \xi_H(t)$ for sufficiently small t when $\lambda_0 > \lambda_1$. Then, the first term is negative as $P'_L(t) < 0$. Also note that by $\mu/2 > K$, $T^* \rightarrow \infty$ as $\sigma \rightarrow \infty$. Therefore, as $\sigma \rightarrow \infty$, the value of (3) is lower than

$$\int_{-\infty}^{\infty} P'_H(t) \frac{\xi_H(t)}{\xi_H(\hat{t})} dt + [P_H(\hat{t}) - P_L(\hat{t})].$$

As $[P_H(\hat{t}) - P_L(\hat{t})]$ is independent of σ , and $\xi_H(t)/\xi_H(\hat{t}) \rightarrow \infty$, the above equation diverges to $-\infty$. This shows that $D < 0$.

Next, consider a low σ case. Since $\Pi_H^*(t)$ can be represented by σ -independent form using $a = \exp(\lambda \sigma^{-2}(t - (\lambda_0 + \lambda_1)/2))$, $T^* = \sigma^2 \lambda^{-1} \ln(a^H) + (\lambda_0 + \lambda_1)/2$ for some a^H , which is independent of σ . Also, since $\hat{t} = \sigma^2 \lambda^{-1} \ln(\hat{a}) + (\lambda_0 + \lambda_1)/2$,

$$T^* - \hat{t} = \sigma^2 \lambda^{-1} \ln(a^H/\hat{a}).$$

Further,

$$\begin{aligned} \frac{\xi_H(t)}{\xi_H(\hat{t})} &= \frac{\mu \cdot \exp\left((\hat{t} - t) \cdot \frac{\hat{t} + t - 2\lambda_1}{2\sigma^2}\right) + (1 - \mu) \cdot \exp\left((\hat{t} - t + \lambda_0 - \lambda_1) \cdot \frac{\hat{t} + t - (\lambda_0 + \lambda_1)}{2\sigma^2}\right)}{\mu + (1 - \mu) \cdot \hat{a}} \\ \frac{\xi_L(t) - \xi_H(t)}{\xi_H(\hat{t})} &= (2\mu - 1) \cdot \frac{\exp\left((\hat{t} - t) \cdot \frac{\hat{t} + t - 2\lambda_1}{2\sigma^2}\right) - \exp\left((\hat{t} - t + \lambda_0 - \lambda_1) \cdot \frac{\hat{t} + t - (\lambda_0 + \lambda_1)}{2\sigma^2}\right)}{\mu + (1 - \mu) \cdot \hat{a}} \end{aligned}$$

Note that both T^* and $\hat{t}$ converge to $(\lambda_0 + \lambda_1)/2$ as $\sigma \rightarrow 0$. Then, if $t \in (\hat{t}, T^*)$,

$$\begin{aligned} 0 &\leq (\hat{t} - t + \lambda_0 - \lambda_1) \cdot \frac{\hat{t} + t - (\lambda_0 + \lambda_1)}{2\sigma^2} < \ln(\hat{a}) + \ln(a^H) \\ (\hat{t} - t) \cdot \frac{\hat{t} + t - 2\lambda_1}{2\sigma^2} &\rightarrow 0. \end{aligned}$$

Then, $\xi_H(t)/\xi_H(\hat{t})$ converges to a finite value. Thus,

$$\int_{\hat{t}}^{T^*} [-P'_H(t)] \frac{\xi_H(t)}{\xi_H(\hat{t})} dt < [T^* - \hat{t}] \max_t \left[-P'_H(t) \frac{\xi_H(t)}{\xi_H(\hat{t})} \right] \rightarrow 0.$$

Similarly, if $t < \hat{t}$,

$$(\hat{t} - t + \lambda_0 - \lambda_1) \cdot (\hat{t} + t - (\lambda_0 + \lambda_1)) \rightarrow \left(\frac{\lambda_0 + \lambda_1}{2} - t + \lambda \right) \left(t - \frac{\lambda_0 + \lambda_1}{2} \right) < 0$$

Therefore, $(\xi_L(t) - \xi_H(t))/\xi_H(\hat{t})$ converges to a positive value as $\sigma \rightarrow 0$. Using (3), we can conclude that $D > 0$ for sufficiently small σ . ■

Proof of (b): Next, we consider the low μ case: for each $t < T_L(K)$, $\Pi_L^*(t) > \Pi_H^*(t)$. This implies that for any $t < T_L(K)$, the firm sets price $P_L(t)$ and discontinues the product sale if $t > T_L(K)$.

To simplify the notation, we denote $T_* = T_L(K)$. The utility of the type- H from the product sales is

$$w \int^{T_* - d\tau} [P_H(t) - P_L(t + d\tau)] \xi_H(t) dt.$$

The differentiation concerning $d\tau$ is the marginal gain by participation. Therefore, they participate in the protest if and only if

$$c_H := -w[P_H(T_*) - P_L(T_*)]\xi_H(T_*) + w \int^{T_*} [-P'_L(t)] \xi_H(t) dt \geq c_i.$$

Similarly, the utility of the type- L is $w \int^{T_* - d\tau} [P_L(t) - P_L(t + d\tau)] \xi_L(t) dt$. Then, they participate in the protest if and only if

$$c_L := w \int^{T_*} [-P'_L(t)] \xi_L(t) dt \geq c_i.$$

Then, $c_L > c_H$ if and only if $\tilde{D} > 0$, which is defined by

$$\tilde{D} := \int^{T_*} [-P'_L(t)] (\xi_L(t) - \xi_H(t)) dt + [P_H(T_*) - P_L(T_*)] \xi_H(T_*).$$

We complete the proof with the following claim.

Claim 12. Suppose that $\mu \in (\mu^*, \mu^\dagger)$. Then, for sufficiently large σ , $\tilde{D} > 0$.

Proof of Claim 12. Dividing $\tilde{D}$ by $\xi_H(T_*)$ yields that

$$\frac{\int^{T_*} [-P'_L(t)] (\xi_L(t) - \xi_H(t)) dt}{\xi_H(T_*)} + [P_H(T_*) - P_L(T_*)] \quad (4)$$

As $\mu > \mu^*$, $\lim_{\sigma \rightarrow \infty} T_* = -\infty$ by Lemma 4. Therefore, the first term of (4) converges to

$$-P'_L(T_*) \frac{\xi_L(T_*) - \xi_H(T_*)}{\xi'_H(T_*)}.$$

Note that

$$\frac{\xi_L(T_*) - \xi_H(T_*)}{\xi'_H(T_*)} = \frac{(2\mu - 1) \left(1 - \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \left(T_* - \frac{\lambda_1 + \lambda_0}{2}\right)\right) \right)}{\mu \frac{T_* - \lambda_1}{\sigma^2} + (1 - \mu) \frac{T_* - \lambda_0}{\sigma^2} \exp\left(\frac{\lambda_0 - \lambda_1}{\sigma^2} \left(T_* - \frac{\lambda_1 + \lambda_0}{2}\right)\right)}.$$

As shown in Lemma 4, T_*/σ^2 converges to a finite value. Then, the above value converges to a finite value.

Note also that

$$-P'_L(t) = \frac{\lambda_0 - \lambda_1}{\sigma^2} \frac{\mu}{1 - \mu} \times \frac{1}{\frac{1}{a} + 2\frac{\mu}{1-\mu} + \left(\frac{\mu}{1-\mu}\right)^2 a} > 0.$$

As T_*/σ^2 converges to a finite value, a is also a finite value. Then, $-P'_L(T_*) \rightarrow 0$. Therefore, (4) reduced to $[P_H(T_*) - P_L(T_*)]$, which is calculated as

$$P_H(t) - P_L(t) = \frac{\frac{\mu}{1-\mu} - \frac{1-\mu}{\mu}}{\left(1 + \frac{\mu}{1-\mu}a\right)\left(1 + \frac{1-\mu}{\mu}a\right)}a,$$

As a is a finite value, $\lim_{\sigma^2 \rightarrow \infty} P_H(T_*) - P_L(T_*) > 0$. This concludes that $\tilde{D} > 0$. ■

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Online Appendix

B. Omitted proof for standard fact

Proof of Fact 1. Note that ex post belief $\{P_\theta(t)\}_{t \in \mathbb{R}}$ has mean θ , which is independent of λ .

Note also that $P_\theta(t) = \theta / (\theta + (1 - \theta)a)$, where $a = \exp(\lambda \sigma^{-2} \cdot (t - (\lambda_0 + \lambda_1)/2))$. By letting $Z = \ln(\theta(1 - \theta)^{-1}(1 - \pi)\pi^{-1})$,

$$\Pr(P_\theta(t) > \pi \mid \lambda) = \theta \Phi\left(\frac{\lambda}{2\sigma} + \frac{\sigma}{\lambda} Z\right) + (1 - \theta) \Phi\left(\frac{\lambda}{2\sigma} - \frac{\sigma}{\lambda} Z\right)$$

and then,

$$\frac{\partial \Pr(P_\theta(t) > \pi \mid \lambda)}{\partial \lambda} > 0 \iff \frac{\pi - \frac{1}{2}}{\sigma^2} \lambda^2 + \ln\left(\frac{\pi}{1 - \pi}\right) > \ln\left(\frac{\theta}{1 - \theta}\right)$$

As the left-hand side of the above inequality is increasing in π , $\Pr(P_\theta(t) > \pi \mid \lambda)$ has the single-crossing property, which implies that $\Pr(P_\theta(t) > \pi \mid \lambda)$ with a higher λ is a mean-preserving spread of that with a lower λ . ■

C. High-noise case for continuous types

In this section, we analyze the case where σ is sufficiently high, and examine price-informativeness interaction in continuous signal model.

As we discussed in Section 3.2, the effect of p on c_θ are decomposed into the pivotal effect and the information-rent effect as follows:

$$c_\theta(p) = -w \underbrace{[P_\theta(T_f(p)) - p]_+}_{\text{Information rent}} \cdot \underbrace{\xi_\theta(T_f(p))}_{\text{Pivotal prob.}}$$

We show that the pivotal effect dominates the information-rent effect when σ is sufficiently high. First note that, when σ is high, $T_f(p)$ is sufficiently large. Because ξ_θ is the mean of $\varphi((t - \lambda_v)/\sigma)$ and $\varphi'(t') < 0$ for sufficiently large t' , it follows that $\xi'_\theta(t') < 0$. Therefore, the probability of being pivotal increases. This widens the gap in c_θ across different θ . Since the informativeness of the protest depends on the difference in participation cutoffs across θ , λ increases as p increases. The effect on information rent is dominated by the pivotal effect, because the effect on information rent is proportional to ξ_θ , which converges to zero at the rate $\sigma^{-1} \exp(-\sigma)$.

The following theorem provides a general condition for the high-noise case.

Theorem 4. $\partial \Lambda(\lambda, p) / \partial p > 0$ for sufficiently large σ if and only if $[\partial a^*(p) / \partial p] \cdot \ln(a^*(p)) < 0$.

Now we consider the optimal price choice in a high-noise environment. Let $p_\sigma^\dagger$ be the optimal price when the noise level is σ when the effect on λ is ignored.

As Lemma 2 shows, in a high-noise environment (i.e., $\sigma \rightarrow \infty$), we have $\lim_{\sigma \rightarrow \infty} p_\sigma^\dagger = p_1$; that is, the price maximizes $\Pi^*(p, 1)$. The case $a = 1$ corresponds to $t = 0$, which implies no information is obtained from the protest. If the firm prefers to sell a product (i.e., $\Pi^*(p_1, 1) > 0$), then $a^*(p_1) > 1$. Therefore, by Theorem 4, $\partial \Lambda(\lambda, p_1)/\partial p_1 > 0$ for sufficiently large σ if and only if $\partial a^*(p_1)/\partial p_1 < 0$. $\partial a^*(p)/\partial p < 0$ is satisfied, for example, if $\partial^2 \Pi^*(p, a)/\partial p \partial a < 0$. We provide a sufficient condition below. In this case, price and informativeness are complements: the optimal price should be higher when the firm prefers more informative protests.

Proposition 6. *Suppose that $g_1(\theta) = 2\theta$ and $g_0(\theta) = 2(1 - \theta)$. Then, if $a^*(p_1) \approx 1$ and $\Pi^*(p_1, 1) > 0$, $\lim_{\sigma \rightarrow \infty} \partial \Lambda(\lambda, p_\sigma^\dagger)/\partial p > 0$.*

C.1. Proofs

Proof of Theorem 4. We first consider the case with a high σ .

Note that we write that for any function of σ , $x(\sigma)$ and $y(\sigma)$, $x \propto y$ if $\lim_{\sigma \rightarrow \infty} |x(\sigma)/y(\sigma)| \in \mathbb{R}_{++}$. We prepare the following two claims.

Claim 13. *As $\sigma \rightarrow \infty$, $|\xi_\theta(T_f(p))| \propto \sigma^{-1}e^{-\sigma}$ and $(\xi'_\theta(T_f(p)))/(\xi_\theta(T_f(p))) \rightarrow -\ln(a^*(p))$.*

Proof of Claim 13.

$$\xi'_\theta(T_f(p)) = -\frac{1}{\sigma} \varphi\left(\frac{T_f(p) - \lambda_1}{\sigma}\right) \left[\theta \frac{T_f(p) - \lambda_1}{\sigma^2} + (1 - \theta) \frac{T_f(p) - \lambda_0}{\sigma^2} a \right],$$

$$T_f(p) - \lambda_1 = \frac{\lambda}{2} + \frac{\sigma^2}{\lambda} \ln(a^*(p)), \text{ and } T_f(p) - \lambda_0 = -\frac{\lambda}{2} + \frac{\sigma^2}{\lambda} \ln(a^*(p)).$$

$$\xi_\theta(T_f(p)) = \frac{1}{\sigma} \varphi\left(\frac{T_f(p) - \lambda_1}{\sigma}\right) [\theta + (1 - \theta)a],$$

Then, $\lim_{\sigma \rightarrow \infty} \frac{\xi'_\theta(T_f(p))}{\xi_\theta(T_f(p))} = -\ln(a^*(p))$. ■

The next claim shows convergence rates of the derivatives of c_θ .

Claim 14. *For sufficiently large σ ,*

$$\text{sign}\left(\frac{\partial^2 c_\theta(p)}{\partial \theta \partial p}\right) = \text{sign}\left(\frac{\partial a^*(p)}{\partial p} \ln(a^*(p))\right).$$

Proof of Claim 14. As in the proof of Claim 2, the cross derivative is calculated as

$$\frac{\partial^2 c_\theta(p)}{\partial \theta \partial p} = -\xi'_\theta(T_f(p))T'_f(p)Y(p) - \xi_\theta(T_f(p))Y'(p),$$

$$Y(p) = \frac{wa^*(p)}{\theta + (1-\theta)a^*(p)} \left[\frac{\theta}{\theta + (1-\theta)a^*(p)} + \frac{p(1-a)}{a} \right].$$

As $\theta/(\theta + (1-\theta)a) > p$ and $(1-a)/a > -1$, $Y(p) > 0$. As $\lim_{\sigma \rightarrow \infty} \frac{\xi'_\theta(T_f(p))}{\xi_\theta(T_f(p))} = -\ln(a^*)$, for sufficiently large σ ,

$$T'_f(p) = \frac{\sigma^2}{\lambda} \frac{\partial a^*(p)}{\partial p}, \text{ and } \text{sign}\left(\frac{\partial^2 c_\theta(p)}{\partial \theta \partial p}\right) = \text{sign}\left(\frac{\partial a^*(p)}{\partial p} \ln(a^*(p))\right).$$

■

Finally, we determine the sign of $\partial \Lambda(\lambda, p)/\partial p$. Note that

$$\begin{aligned} \frac{\partial \Lambda(\lambda, p)}{\partial p} &= \int_{\theta(p)} f(c_\theta(p)) \frac{\partial c_\theta(p)}{\partial p} (g_0(\theta) - g_1(\theta)) d\theta \\ &= -f(c_{\theta(p)}(p)) \frac{\partial c_{\theta(p)}(p)}{\partial p} (G_0(\theta(p)) - G_1(\theta(p))) \\ &\quad - \int_{\theta(p)} \left[f(c_\theta(p)) \frac{\partial^2 c_\theta(p)}{\partial \theta \partial p} + f'(c_\theta(p)) \frac{\partial c_\theta(p)}{\partial \theta} \frac{\partial c_\theta(p)}{\partial p} \right] (G_0(\theta) - G_1(\theta)) d\theta \end{aligned}$$

Claim 13 implies that $\xi_\theta(T_f(p)) \rightarrow 0$ and $\xi'_\theta(T_f(p))/\xi_\theta(T_f(p)) \rightarrow -\ln(a^*(p))$ as $\sigma \rightarrow \infty$.

Furthermore, we can see that $T'_f(p) \rightarrow \infty$ as $\sigma \rightarrow \infty$. By Claim 2, these facts imply that

$$\lim_{\sigma \rightarrow \infty} \frac{x}{\frac{\partial^2 c_\theta(p)}{\partial \theta \partial p}} \rightarrow 0 \text{ for each } x \in \left\{ \frac{\partial c_\theta(p)}{\partial \theta} \frac{\partial c_\theta(p)}{\partial p}, \frac{\partial c_{\theta(p)}(p)}{\partial p} \right\}.$$

Therefore,

$$\frac{\partial \Lambda(\lambda, p)}{\partial p} \frac{\partial^2 c_\theta(p)}{\partial \theta \partial p} < 0$$

for sufficiently large σ . Then, by Claim 14, the desired result holds. ■

Proof of Proposition 6. By the first-order condition, at $p = p_a$,

$$\frac{\partial \Pi^*(p, a)}{\partial p} = \frac{1}{1+a} (1 - \tilde{G}_1 + a(1 - \tilde{G}_0)) - \frac{p - \kappa}{1+a} (\tilde{g}_1 + a\tilde{g}_0) \frac{1}{p(1-p)} = 0$$

Therefore, at $p = p_1$,

$$\frac{p - \kappa}{p(1-p)} = \frac{2 - \tilde{G}_1 - \tilde{G}_0}{\tilde{g}_1 + \tilde{g}_0}$$

On the other hand,

$$(1+a) \frac{\partial^2 \Pi^*(p, a)}{\partial p \partial a} = \frac{1}{1+a} (\tilde{G}_1 - \tilde{G}_0) - \frac{1}{2a} (\tilde{g}_1 + a\tilde{g}_0) + \frac{p-\kappa}{p(1-p)} \left[\frac{\tilde{g}_1 - \tilde{g}_0}{1+a} - \frac{\tilde{g}'_1 + a\tilde{g}'_0}{2a} \right].$$

Therefore, $\partial^2 \Pi^*(p, a) / \partial p \partial a < 0$ at $a = 1$ and $p = p_1$ if

$$A := \frac{1}{2} (\tilde{G}_1 - \tilde{G}_0) - \frac{1}{2} (\tilde{g}_1 + \tilde{g}_0) + \frac{2 - \tilde{G}_1 - \tilde{G}_0}{\tilde{g}_1 + \tilde{g}_0} \left[\frac{\tilde{g}_1 - \tilde{g}_0}{2} - \frac{\tilde{g}'_1 + \tilde{g}'_0}{2} \right] < 0.$$

Now suppose that $g_1(\theta) = 2\theta$ and $g_0(\theta) = 2(1 - \theta)$. Then, the above equation is calculated as follows.¹⁸

$$A = -\theta(1 - \theta) - 1 + 2(1 - \theta)(2\theta - 1) < 0,$$

where $\theta = \psi(\omega(p, a)) \in [0, 1]$ at $p = p_1$, and $a = 1$.

Therefore, $\partial \Pi^*(p, a) / \partial p < 0$ for $a \in (1, 1 + \varepsilon)$ for small $\varepsilon > 0$ at $p = p_1$. Furthermore, by the assumption that $\Pi^*(p_1, 1) > 0$, $a^*(p_1) > 1$ and this is sufficiently close to 1.¹⁹ Then, as $\partial \Pi^*(p, a) / \partial a < 0$, $\partial a^*(p) / \partial p < 0$ at $p = p_1$.

By Theorem 4, $\partial \Lambda(\lambda, p) / \partial p > 0$. ■

D. Profit and Informativeness

This section analyzes the relationship between profit and informativeness in the continuous signal model. We show that, when σ is sufficiently small, the firm prefers more informative protests.

To this end, we calculate the firm's expected profit as follows:

$$\begin{aligned} \Pi^* &:= \int_{-\infty}^{T_f(p)} \Pi(p, t) \left[\varphi\left(\frac{t-\lambda_1}{\sigma}\right) + \varphi\left(\frac{t-\lambda_0}{\sigma}\right) \right] dt \\ &= \int_{-\infty}^{\frac{\sigma^2}{\lambda} \ln(a^*(p))} \left[(p - \kappa) \sum_{i \in \{0,1\}} \varphi\left(\frac{t-\lambda_i}{\sigma}\right) (1 - \tilde{G}_i(\omega(p, a))) - K \left[\varphi\left(\frac{t'+\frac{\lambda}{2}}{\sigma}\right) + \varphi\left(\frac{t'-\frac{\lambda}{2}}{\sigma}\right) \right] \right] dt' \end{aligned}$$

where $t' = t - (\lambda_0 + \lambda_1)/2$ and $a = \exp(t' \lambda / \sigma^2)$. Note that the change of variable eliminates dependence on $\lambda_0 + \lambda_1$, so Π^* depends on λ_0 and λ_1 only through $\lambda = \lambda_0 - \lambda_1$. By differentiating the above equation with respect to λ , we have

¹⁸ Note that $\tilde{G}_v(x) = G_v(\psi(x))$. Then, as g_v is linear, $\tilde{g}_v = g_v \psi'$ and $\tilde{g}'_v = g_v \psi''$. Recall that $\psi = \exp(x)/(1 + \exp(x))$. Now we have $\psi''/\psi' = (1 - \exp(x))/(1 + \exp(x)) = (1 - 2\theta)$ by letting $\theta = \exp(x)/(1 + \exp(x))$.

¹⁹ This is possible by taking K adequately.

$$\begin{aligned} \frac{\partial \Pi^*}{\partial \lambda} = & \int_{-\infty}^{\frac{\sigma^2}{\lambda} \ln(a^*(p))} \left[-(p - \kappa) \sum_{i \in \{0,1\}} \varphi\left(\frac{t - \lambda_i}{\sigma}\right) \left(2\tilde{G}'_i(\omega(p, a)) \frac{t'}{\sigma^2} \right) \right. \\ & \left. + \frac{1}{\sigma} \left[((1 - \tilde{G}_1(\omega(p, a))) - K) \varphi'\left(\frac{t' + \frac{\lambda}{2}}{\sigma}\right) - ((1 - \tilde{G}_0(\omega(p, a))) - K) \varphi'\left(\frac{t' - \frac{\lambda}{2}}{\sigma}\right) \right] \right] dt'. \end{aligned}$$

Proposition 7. *For sufficiently small σ , $\partial \Pi^* / \partial \lambda > 0$ whenever $\lambda > 0$.*

Proof of Proposition 7. As $a^*(p)$ is independent of σ , the upper limit of integration $\sigma^2 / \lambda \ln(a^*(p)) \rightarrow 0$ as $\sigma \rightarrow 0$, so the domain of integration converges to $(-\infty, 0)$.

To evaluate the sign of $\partial \Pi^* / \partial \lambda$ as $\sigma \rightarrow 0$, we focus on the first term of the integrand, which dominates. Factor out $1/\sigma^2$ and consider the rescaled integrand. The normal densities $\varphi((t' + \lambda/2)/\sigma)$ and $\varphi((t' - \lambda/2)/\sigma)$ concentrate at $t' = -\lambda/2$ and $t' = \lambda/2$, respectively, with scale σ . Since the upper limit of integration converges to 0 and $\lambda > 0$, only $t' = -\lambda/2 < 0$ lies in the interior of the domain.

Therefore, the dominant contribution as $\sigma \rightarrow 0$ comes from the first term evaluated near $t' = -\lambda/2$:

$$\frac{\partial \Pi^*}{\partial \lambda} \approx -(p - \kappa) \varphi(0) \cdot \sum_{i \in \{0,1\}} \tilde{G}'_i(\omega(p, a^{-1/2})) \cdot \frac{-\lambda}{\sigma^2}.$$

Since $\tilde{G}'_i > 0$ (as $\tilde{G}_i$ is a distribution function) and $t' = -\lambda/2 < 0$, the product $\tilde{G}'_i \cdot t'$ is negative. Thus, the overall expression is positive whenever $\lambda > 0$. ■

This result, combined with Proposition 1, reveals a fundamental tension: in low-noise environments, the firm benefits from more informative protests (as they enable better discontinuation decisions), yet its profit-maximizing price endogenously destroys this informativeness through the information-rent effect. The firm's private incentive to extract surplus thus conflicts with its own interest in acquiring information from collective action.